INTELLIGENT OPTIMIZATION OF SOLAR-FOSSIL HYBRID REFRIGERATION SYSTEMS USING ANN SURROGATE MODELS AND METAHEURISTIC ALGORITHMS

OPTIMIZACIÓN INTELIGENTE DE SISTEMAS DE REFRIGERACIÓN HÍBRIDOS SOLAR-FÓSIL MEDIANTE MODELOS SUSTITUTOS CON REDES NEURONALES ARTIFICIALES Y ALGORITMOS METAHEURÍSTICOS

Víctor Cardoso Fernández
Universidad Autónoma de Yucatán, México
victor.cardoso@alumnos.uady.mx

Luis Josué Ricalde Castellanos
Universidad Autónoma de Yucatán, México
lricalde@correo.uady.mx

Bassam Ali
Universidad Autónoma de Yucatán, México
baali@correo.uady.mx

Mauricio Alberto Escalante Soberanis
Universidad Autónoma de Yucatán, México
mauricio.escalante@correo.uady.mx

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Abstract

This study presents an optimized hybrid refrigeration system powered by solar energy and natural gas, aimed at improving energy efficiency, economic performance, and environmental sustainability. The methodology combines thermoenergetic analysis, climatic adaptation across various Mexican regions, and advanced tools such as artificial neural networks and metaheuristic algorithms (Particle Swarm Optimization and TOPSIS). Through climate-informed optimization, the system is tailored to specific environmental conditions, enhancing performance. Results show significant improvements in Net Present Value and carbon emissions reduction, especially in semi-arid climates. The integration of intelligent algorithms enables the design of context-sensitive refrigeration solutions, demonstrating the

potential of hybrid systems to meet cooling demands while minimizing environmental impacts. This work highlights the importance of localized adaptations and strategic optimization for advancing renewable energy integration in thermal systems, offering a replicable model for sustainable cooling applications in diverse regions.
Keywords: Computational intelligence, hybrid refrigeration systems, metaheuristic optimization, solar-powered cooling, thermoenergetic analysis.

Resumen

Este estudio presenta un sistema de refrigeración híbrido optimizado, alimentado por energía solar y gas natural, orientado a mejorar la eficiencia energética, el desempeño económico y la sostenibilidad ambiental. La metodología combina análisis termoenergético, adaptación climática en diversas regiones de México y herramientas avanzadas como redes neuronales artificiales y algoritmos metaheurísticos (Optimización por Enjambre de Partículas y TOPSIS). A través de una optimización basada en condiciones climáticas, el sistema se adapta a entornos específicos, mejorando su rendimiento. Los resultados muestran mejoras significativas en el Valor Presente Neto y la reducción de emisiones de carbono, especialmente en climas semiáridos. La integración de algoritmos inteligentes permite diseñar soluciones de refrigeración sensibles al contexto, demostrando el potencial de los sistemas híbridos para satisfacer la demanda de enfriamiento minimizando los impactos ambientales. Este trabajo resalta la importancia de las adaptaciones localizadas y la optimización estratégica para impulsar la integración de energías renovables en sistemas térmicos, ofreciendo un modelo replicable para aplicaciones sustentables de refrigeración.
Palabras Clave: *Análisis termoenergético, inteligencia computacional, optimización metaheurística, refrigeración solar, sistemas de refrigeración híbridos.*

1. Introduction

The imperative for sustainable and energy-efficient solutions in the contemporary world is underscored by escalating environmental concerns and the urgent global need to combat climate change. Among various technological innovations, solar-powered cooling systems are increasingly recognized for their potential to transform living environments into smarter, more sustainable spaces while significantly reducing carbon emissions and decreasing reliance on fossil fuels [Al-Yasiri, 2022; Rashid, 2023; Jiao, 2023].

In the pursuit of advancing smart living paradigms, the integration of solar energy with refrigeration systems plays a pivotal role. Smart living, in the context of modern urban development, refers to the use of integrated and automated systems to enhance the efficiency and sustainability of residential and commercial environments. This approach leverages renewable energy technologies to ensure that energy usage is both efficient and minimal, thereby fostering environments that are not only technologically advanced but also environmentally responsible and economically viable. Recent research has increasingly focused on the development of hybrid cooling systems that combine the benefits of solar energy with the reliability of conventional fossil fuels. These systems address sustainability challenges while ensuring consistent performance across diverse environmental conditions. For example, studies exploring the integration of solar collectors with traditional absorption refrigeration systems highlight the ability of solar thermal energy to drive the cooling process, supplemented by fossil fuel heaters to guarantee continuous operation under variable conditions [Jenkins, 2020; Herrando, 2019].

Moreover, the adoption of hybrid photovoltaic-thermal (PVT) collectors alongside vapor compression systems represents a significant innovation, enabling the simultaneous generation of electricity and thermal energy for cooling purposes. This dual-functionality not only improves overall energy efficiency but also enhances the resilience and flexibility of cooling systems, thereby contributing to smarter energy management within built environments [Settino, 2023; Rosales, 2023].

The refinement of these hybrid systems through metaheuristic optimization algorithms underscores a promising avenue for enhancing system performance and efficiency. Techniques such as particle swarm optimization (PSO), genetic algorithms (GA), and Simulated Annealing (SA) are particularly effective in navigating the complex, nonlinear optimization challenges inherent in hybrid energy systems. These advanced computational methods facilitate the design and operation of optimized configurations that maximize energy output, reduce costs, and improve overall system reliability, thereby aligning with smart living objectives [Tseng, 2021; Lei, 2020; Alnowibet, 2022]. Hybrid refrigeration systems, which seamlessly integrate solar technology with conventional energy sources, are ideally suited to meet the cooling demands of modern urban lifestyles in residential, commercial, and industrial sectors. By harnessing solar energy, these systems not only curtail reliance on finite fossil resources but also mitigate broader environmental impacts, notably greenhouse gas emissions. However, optimizing the performance of such complex systems requires a careful balance of thermodynamic processes, economic factors, and environmental considerations [Duda, 2019; Jain, 2019].

To comprehensively address these challenges, this study proposes a novel methodology that includes three distinct stages: thermoenergetic analysis of the hybrid Generator-Absorber Heat Exchange (GAX) cycle, the development of surrogate models using artificial intelligence techniques, and the application of metaheuristic algorithms for detailed optimization and feasibility analysis. These methodologies are intricately designed to systematically analyze, model, and optimize the hybrid refrigeration system, targeting enhanced energy efficiency, economic viability, and environmental sustainability.

The innovation of this approach lies in its comprehensive methodology that not only aims to optimize hybrid refrigeration systems powered by solar energy and fossil fuels but also contributes to the broader discourse on smart living. By integrating advanced thermoenergetic analysis, artificial intelligence modeling, and robust metaheuristic optimization techniques, our study provides novel insights into the design, operation, and optimization of hybrid cooling systems, thereby advancing the frontiers of smart living and contributing to the development of innovative solutions for a more sustainable and resilient future.

2. Methods

The methodology presented in this study consists of three main stages. The first stage involves a thermoenergetic analysis of the hybrid GAX cycle with solar integration. The second stage focuses on building an artificial neural network to predict system performance. Finally, the third stage applies particle swarm optimization to optimize the system's parameters, enhancing different indicators.

Stage 1: Thermoenergetic analysis of the hybrid GAX cycle

The methodology begins with the thermoenergetic analysis of (GAX) cycle integrated with solar technology. This initial stage involves assessing the environmental conditions and the system's parameters and independent variables. These elements are crucial for understanding the baseline performance of the GAX cycle under varying climatic and operational conditions. Indicators, specifically the $4\,E$ indicators (energy, economy, environment, and exergy), are derived from this analysis to evaluate the system's performance comprehensively. Separate mathematical models for flat plate (FPC) and parabolic trough (PTC) solar collectors are used to predict how efficiently each technology converts sunlight into thermal energy, which is then used in the refrigeration cycle. The results from these models are stored in a database, which serves as the foundation for AI-based performance predictions and further optimization in later stages.

Stage 2: Surrogate model using artificial intelligence techniques

In this stage, a surrogate model is constructed using artificial neural Network techniques. The ANN architecture is manually inputted, reflecting the specific dynamics and interactions within the hybrid GAX cycle. The ANN model undergoes rigorous training, testing, and validation phases. If the coefficient of determination (R^2) from the validation exceeds 0.99, indicating high predictive accuracy, the model is approved for further use; otherwise, adjustments are made to the architecture or data inputs. Once validated, the ANN model helps identify which independent variables most effectively optimize the $4\,E$ performance indicators, providing critical insights into how changes in system parameters affect overall performance.

Stage 3: Optimization and feasibility analysis

The final stage involves the application of a metaheuristic optimization algorithm, specifically particle swarm optimization, to refine the system configurations for optimal performance based on the insights gained from the ANN model. This optimization process aims to fine-tune the system by adjusting the independent variables in a way that maximizes energy efficiency, cost-effectiveness, environmental sustainability, and system exergy, thereby aligning with the $4\,E$ indicators. The outcome of this stage is a set of optimized system parameters that enhance the feasibility and overall efficacy of the hybrid GAX refrigeration system, ensuring that it meets the desired performance benchmarks in a real-world application.

Thermoenergetic analysis of the hybrid GAX cycle

The GAX cycle, comprising the generator, absorber, condenser, and evaporator, forms the foundation of the hybrid refrigeration system under investigation. Each component plays a crucial role in the thermodynamic processes involved in energy transfer and refrigerant circulation, ultimately contributing to the overall efficiency and performance of the system.

The cycle begins at the output of the rectifier, where ammonia (NH_3) vapor at high system pressure is directed to the condenser with a 99% purity. In the condenser, the vapor is converted into saturated liquid and sent to the pre-cooler to lower its temperature. The fluid then passes through a pressure valve, turning into a low-pressure liquid-vapor mixture. In the evaporator, this mixture absorbs heat from the water that is being cooled, thereby generating the refrigeration effect for the intended application. After leaving the evaporator, the mixture moves back through the pre-cooler, which acts as a heat exchanger, utilizing the heat from the opposing flow to evaporate any remaining liquid in the mixture. This cooled vapor then enters the bottom of the absorber, where it is absorbed by a dilute NH_3/H_2O solution. Heat exchangers are required in this area of the absorber due to the exothermic phase changes that occur. The aqueous solution, now enriched with a higher percentage of the refrigerant, is pumped at high pressure to the next component of the absorber (AHX), where it absorbs heat from the absorber. Entering the hotter element (GAX), the solution receives even more heat from the absorber, reaching its saturation point as it exits the column as a liquid-vapor mixture. This mixture enters a chamber located between the generator and the rectifier, where the liquid phase from the absorber contacts the condensed vapor from the rectifier. Once in the generator, the refrigerant is extracted from the solution, turning it into a weak solution that exits from the bottom of the generator's column. The solution is heated in the generator by external heat sources (in this case, natural gas and solar thermal energy) and transfers this energy into the GHX section. The second pressure valve returns it to low pressure, enabling its reintegration into the top of the absorber, where it meets the refrigerant vapor. Meanwhile, at the bottom of the generator, the refrigerant vapor rises through the column to reach the rectifier, where heavier elements are condensed. This results in a highly pure refrigerant, ready to restart the cycle.

$4E$ indicators

The $4E$ analysis constitutes a comprehensive evaluation of the hybrid refrigeration system, encompassing energy, economy, environment, and exergy considerations. The flat plate solar collector, serving as a key component for supplying thermal energy to the hybrid refrigeration system, is governed by a set of mathematical equations describing its operation and performance. These equations encompass parameters such as solar radiation, absorber plate temperature, fluid flow rate, and heat transfer coefficients, which collectively dictate the efficiency and effectiveness of the collector in harnessing solar energy.

As for the energy indicator seen in Equation 1, the mathematical formulation for quantifying the thermal energy generated by the FPC is [Cetina, 2023]:

$$ETH_{FPC} = A_c[G\tau\alpha - UL(T_{in} - T_{amb})] \qquad (1)$$

Where ETH_{FPC} is the thermal energy output of the flat plate solar collector (J), A_c is the area of the colector (m^2), G is the solar irradiation $\left(\frac{kWh}{m^2}\right)$, τ is the transmittance of the covering (%), α is the absorptivity of the absorbing plate (%), UL is the loss coefficient factor (W/m^2K), T_{in} and T_{amb} are the absorber plate temperature and the ambient temperature, respectively (K).

Additionally, the parabolic trough solar collector enables a broader capture and conversion of solar energy into thermal energy. The PTC, designed to focus sunlight onto a receiver tube running along the focal line of the parabolically curved reflector, enhances the system's thermal efficiency. The useful heat generated by the PTC collector is described by Equation 2.

$$ETH_{PTC} = A_{PTC}\eta_{opt}I\eta_{thermal} \tag{2}$$

Where ETH_{PTC} is the useful thermal energy output (J), A_{PTC} is the aperture area of the trough (m^2), η_{opt} is the optical efficiency of the collector, I is the solar irradiation (kWh/m^2), and $\eta_{thermal}$ is the thermal efficiency, which accounts for heat losses and the heat transfer properties of the fluid within the receiver tube. This equation provides a framework for evaluating the PTC collector's contribution to the system's energy output, complementing the flat plate collector's capabilities.

The net present value calculation is fundamental to assessing the economic viability of integrating solar technology in refrigeration systems [Cetina, 2021], Equation 3.

$$NPV = \sum_{t=1}^{n} \frac{CF_t}{(1+IRR)^t} - I_0 \tag{3}$$

Where NPV represents the net present value in USD, IRR is the internal rate of return, CF_t represents the annual cash flow in USD, and I_0 is the initial investment. This indicator evaluates the long-term financial benefits against the upfront costs, factoring in the operational savings from using solar energy.

For the environmental indicator seen in Equation 4, the amount of carbon mitigated (CCM) is used to quantify the environmental impact in terms of carbon dioxide emissions saved due to the hybrid system's operation [May, 2020].

$$CCM = 3.664\frac{E_{th}CC_{fl}}{\eta_{fl}CV_{fl}} \tag{4}$$

Where CCM measures the CO_2 saved (kg), E_{th} is the solar field annual energy contribution (E_{th}), CC_{fl} is the fuel carbon content $(kg \cdot C \cdot {kg_{fl}}^{-1})$, CV_{fl} is the fuel calorific value $(kWh \cdot {kg_{fl}}^{-1})$, η_{fl} is the efficiency of the backup boiler and 3.664 is the stoichiometric coefficient for total carbon oxidation. This indicator not only reflects the direct reduction in fossil fuel use but also the enhancement in environmental sustainability through reduced greenhouse gas emissions.

For the exergy indicator seen in Equation 5, the irreversibility within the system, or exergy destruction (IRR), is calculated to understand the thermodynamic inefficiencies [Koholé, 2022].

$$IRR = \sum Ex_{loss} + \sum Ex_{dest} \tag{5}$$

Where IRR represents the irreversibilites measured at the control volume (J), $\sum Ex_{loss}$ and $\sum Ex_{dest}$ is the summation of the loss and destructed exergy (J), respectively. This indicator helps in identifying areas within the system where energy efficiency can be improved by minimizing energy degradation and losses.

Climatic analysis of Mexican locations

To ensure the robustness of the hybrid GAX refrigeration system, a climatic analysis was conducted across seven cities in México, representing three major climate regions: Tropical, Arid, and Temperate, based on the Köppen climate classification. These cities include:

- Tropical: Villahermosa, Tabasco (Af - Tropical rainforest), and Cancún, Quintana Roo (Aw - Tropical savanna).
- Arid: Ciudad Juárez, Chihuahua (BW - Cold desert), and México City (BS - Cold semi-arid).
- Temperate: Xalapa, Veracruz (Cf - Humid subtropical), Nogales, Sonora (Cs - Mediterranean summer), and Querétaro (Cw - Subtropical highland).

The environmental conditions of these cities, such as temperature and solar radiation, were incorporated into the system's model to evaluate the performance of the GAX cycle under varying climate zones.

Surrogate model using artificial intelligence techniques

The use of a surrogate model is justified by the high computational cost of repeatedly solving the thermoenergetic model during optimization. An ANN was selected because it captures nonlinear relationships, reduces evaluation time, and facilitates multi-objective analysis.

The input layer includes climate type, fuel type, number of collectors, and mass flow rate, while the outputs are the four $4\ E$ indicators. After testing several configurations, the final architecture consisted of a multilayer perceptron with one hidden layer of 50 neurons, tansig activation, and linear output. Training employed the Levenberg–Marquardt algorithm with early stopping, $L2$ regularization, and z-score normalization, validated by 5-fold cross-validation.

Model accuracy was evaluated using R^2, Pearson correlation (r), root mean square error $(RMSE)$ and mean absolute error (MAE). Results showed $R^2 > 0.99$ across training, testing, and validation, supported by high r values and low errors. In addition, hypothesis tests were performed to determine the statistical significance of the correlations and residual errors. Correlation tests yielded p-values < 0.01, confirming that the associations between predicted and observed values are statistically significant, while residual mean tests indicated that prediction errors were unbiased. These outcomes validate the ANN surrogate as a robust and reliable model for integration into the optimization stage.

Optimization and feasibility analysis

A constricted, global-best Particle Swarm Optimization (PSO) was adopted to tune the independent variables, including the number of collectors (NC), mass flow rate (MR), fuel type, and climate, against the multi-criteria objectives. Particle positions represent candidate system configurations, while velocities are updated under a stabilized PSO dynamic that ensures exploration without swarm explosion [Gad, 2022; Sengupta, 2018].

The main hyperparameters and update settings considered for PSO are the following:

- Inertia weight (w): linearly decreased from 0.9 to 0.4 across iterations.
- Cognitive and social coefficients: $c_1 = c_2 = 1.5$.
- Constriction factor: $\chi = 0.729$, preventing velocity explosion and improving convergence stability.
- Velocity clamping: $|v| \leq 20\%$ of each variable's admissible range.
- Topology: global-best with stochastic neighborhood sampling.

- Swarm size: 100 particles; maximum iterations: 100; function tolerance: 1×10^{-4}.

The Pareto diagram, derived from the Pareto principle, is a graphical tool used to visualize the trade-offs between multiple conflicting objectives in a decision-making process. In this study, the Pareto diagram is employed to facilitate multiobjective optimization of the hybrid refrigeration system, enabling the simultaneous consideration of energy, economy, environment, and exergy indicators [Singh, 2021]. The Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) method is a multi-criteria decision-making technique used to determine the best alternative among a set of options based on their proximity to the ideal solution (Equation 6), and furthest from the negative ideal solution, Equation 7 [May, 2020].

$$d_b{}^{+} = \sqrt{\sum_{n=1}^{i} \left(m_{a,b} - m_a{}^{+}\right)^2} \tag{6}$$

$$d_b{}^{-} = \sqrt{\sum_{n=1}^{i} \left(m_{a,b} - m_a{}^{-}\right)^2} \tag{7}$$

Where $m_a{}^{+}$ and $m_a{}^{-}$ are the ideal solution and the negative-ideal solution, respectively, while $m_{a,b}$ is the b point in the Pareto frontier of the a nondimensionalized objective function.

In TOPSIS, each alternative is evaluated against multiple criteria, and its relative closeness to the ideal solution is calculated using distance measures. The alternative with the highest closeness coefficient shown in Equation 8 is considered the best choice:

$$CCO_b = \frac{d_b{}^{+}}{d_b{}^{+} - d_b{}^{-}} \tag{8}$$

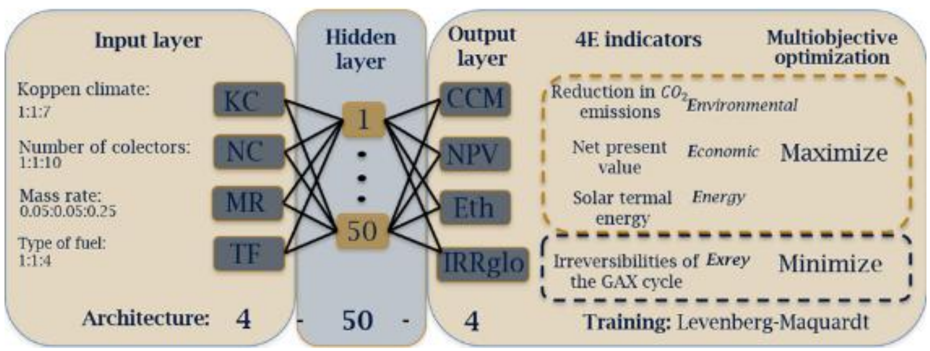
In this study, the optimization and feasibility analysis of the hybrid refrigeration system employs a multifaceted approach combining the genetic algorithm, TOPSIS method, and Pareto diagram to ensure a comprehensive exploration and evaluation of potential solutions. The genetic algorithm, a robust search heuristic, mimics the process of natural selection where solutions evolve iteratively towards optimum by combining exploration and exploitation mechanisms. This algorithm generates a diverse set of potential solutions, which populate the Pareto front, representing the trade-offs among conflicting objectives such as energy efficiency, cost-effectiveness, environmental impact, and system exergy. The Pareto diagram serves as a critical visualization tool in this context, illustrating these trade-offs and highlighting the set of Pareto-optimal solutions where no objective can be improved without worsening another. On this foundation, the TOPSIS method is applied to advance the decision-making process by evaluating each solution against the ideal solution, which maximizes the desired attributes, and the anti-ideal solution, which minimizes them. This evaluation is based on the closeness coefficient, which quantifies how close each solution on the Pareto front is to the ideal solution while being farthest from the anti-ideal, effectively ranking the solutions in order of their overall desirability.

By integrating the Pareto diagram's visual insights with TOPSIS's quantitative analysis, this methodology not only identifies but also ranks optimal solutions, enabling decision-makers to select the most appropriate configuration that achieves a balanced compromise across all evaluated criteria. This comprehensive approach ensures that the final selected solution embodies the best possible combination of performance metrics, aligned with the system's multifaceted objectives.

3. Results

The diagram shown in Figure 1 provides a visual representation of the artificial neural network architecture utilized to optimize the hybrid GAX cycle refrigeration system, focusing on the integration of solar energy and advanced computational techniques to enhance system performance. The input layer of the ANN model consists of several variables critical to the system's operation, which include the Köppen climate classification (KC), the number of collectors (NC), the mass flow rate (MR), and the type of fuel (TF). The input data ranges and options reflect the variability and adaptability required for diverse geographic and climatic conditions, from tropical to cold climates, and different energy sources from natural gas to diesel. The model features a single hidden layer with 50 neurons, indicating a complex pattern processing capability between the input variables and the output performance metrics. The output layer includes four primary performance indicators relevant to the hybrid system's efficiency and sustainability. The ANN is trained using the Levenberg-Marquardt algorithm, a robust method known for its efficiency in handling complex datasets and convergence reliability.

The performance of the ANN surrogate for the hybrid GAX cycle refrigeration system was evaluated across training, testing, and validation phases, as summarized in Table 1. Model accuracy was assessed using the coefficient of determination (R^2), correlation coefficient (r), RMSE and MAE.



Source: own elaboration.

Figure 1 ANN architecture constructed for the hybrid refrigeration system.

Table 1 Performance metrics of the ANN for training, testing, validation, and all data.

Stage	$ETH\ (R^2/r/RMSE/MAE)$	$CCM\ (R^2/r/RMSE/MAE)$	$NPV\ (R^2/r/RMSE/MAE)$	$IRR\ (R^2/r/RMSE/MAE)$
Training	0.99998 /0.9999 /0.0021 /0.0015	0.99994 /0.9999 /0.0034 /0.0027	0.99995 /0.9999 /0.0042 /0.0030	0.99995 /0.9999 /0.0026 /0.0019
Testing	0.98408 /0.9921 /0.0315 /0.0248	0.99379 /0.9962 /0.0197 /0.0142	0.97897 /0.9910 /0.0364 /0.0271	0.99683 /0.9981 /0.0153 /0.0108
Validation	0.99397 /0.9970 /0.0224 /0.0165	0.97795 /0.9899 /0.0386 /0.0291	0.98059 /0.9905 /0.0358 /0.0267	0.99788 /0.9989 /0.0139 /0.0095
All	0.99756 /0.9987 /0.0147 /0.0102	0.99562 /0.9978 /0.0179 /0.0128	0.99472 /0.9973 /0.0192 /0.0137	0.99906 /0.9995 /0.0096 /0.0067

Source: own elaboration.

During training, the ANN achieved near-perfect results. Testing with unseen data showed small but expected reductions, particularly for Solar thermal energy (ETH)

and Net present value (NPV), while validation confirmed robust predictive power across all indicators with $R^2 > 0.97$, high correlation values, and low errors.

The combined dataset yielded overall R^2 values above 0.99, with consistently strong r and low $RMSE$ and MAE across indicators, confirming the ANN's reliability and generalization capacity for integration into the optimization stage.

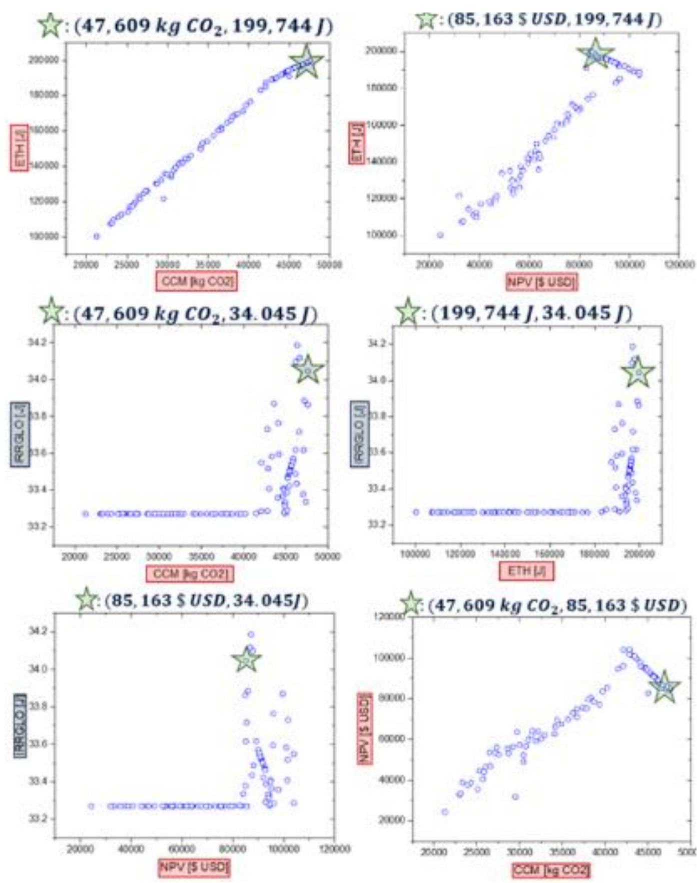
The particle swarm optimization algorithm was configured with parameters designed to ensure both stability and effective exploration of the solution space for optimizing the $4\,E$ indicators. A swarm size of 100 particles and a maximum of 100 iterations were defined, with a convergence tolerance of 1×10^{-4} acting as an early stopping criterion. The inertia weight decreased linearly from 0.9 to 0.4 across iterations, while cognitive and social coefficients were set to $c_1 = c_2 = 1.5$. A constriction factor of $\chi = 0.729$ and velocity clamping at 20% of the admissible variable range were applied to prevent swarm explosion and premature convergence. The topology used was global-best with stochastic neighborhood sampling. Continuous variables were encoded as real values, while categorical variables such as fuel type and climate were represented through integer or one-hot encoding with repair operators to preserve feasibility. Finally, all variables and objectives were normalized to the range $[0.1, 0.9]$, improving numerical conditioning and enhancing the stability and performance of the optimization process.

The optimization diagrams provided clearly illustrate the relationships between various performance indicators within the multi-objective optimization framework using the TOPSIS method coupled with a genetic algorithm. These diagrams are essential for visualizing how changes in system configurations influence the solar thermal energy output, economic viability, carbon mitigation, and system exergy of the hybrid refrigeration system.

As can be seen from Figure 2, some of the main discussions that can be made from these diagrams are:

- Solar thermal energy (ETH) vs. carbon mitigation (CCM): This plot demonstrates a direct correlation between the amount of solar thermal energy produced (ETH) and the amount of carbon dioxide emissions mitigated (CCM). It highlights that systems which generate more solar thermal energy also contribute more significantly to environmental sustainability by mitigating greater amounts of CO_2.

- Net present value (NPV) vs. solar thermal energy (ETH): The relationship between solar thermal energy output and economic returns is depicted here. It indicates that higher outputs of solar thermal energy tend to correlate with higher net present values, showing that more effective solar energy conversion is not only technically desirable but also economically beneficial.
- System irreversibility (IRR) vs. solar thermal energy (ETH) and carbon mitigation (CCM): These diagrams show how minimizing system irreversibilities can enhance both solar thermal energy output and carbon mitigation. Systems with lower irreversibilities are more efficient, meaning they waste less energy and thereby increase the amount of solar thermal energy utilized while also reducing carbon emissions.



Source: own elaboration

Figure 2 Pareto diagrams created from the combinations between the 6 output variables.

The optimal points highlighted in these plots identify configurations that maximize solar thermal energy production and carbon mitigation while balancing economic returns and minimizing energy losses. These points are key benchmarks for system design, suggesting that a focus on optimizing solar collector performance is essential for achieving superior overall system performance.

Tables 2 and 3 summarizing the optimization results provide a detailed quantitative overview of how different configurations impact the system's performance across various climates. Each entry in these tables corresponds to a specific set of input parameters, including the type and number of collectors, fuel type, and mass rates, offering a detailed look at their effects on the $4\,E$ indicators.

Table 2 Optimized input variables.

Köppen climate (KC)	**Type of fuel** (TF)	**Number of colectors** (NC)	**Mass rate** (MR)
BS: México City	Fuel oil	10	$0.23122\ kg/s$

Source: own elaboration

Table 3 Optimized $4\,E$ indicators.

Energy (Eth)	**Environment** (CCM)	**Economy** (NPV)	**Exergy** (Irr)
$199{,}744.887\ J$	$47{,}609.1301\ kg\ CO_2$	$\$85{,}163.8011\ USD$	$34.045\ J$

Source: own elaboration

4. Discussions

The ANN results confirmed the surrogate’s ability to represent the nonlinear dynamics of the hybrid GAX refrigeration system. High R^2 values across all stages, together with strong correlation coefficients and low $RMSE$ and MAE, validated the model’s robustness. Statistical tests confirmed that the correlations were significant and that residual errors were unbiased, while the slight decline in testing results illustrated its capacity to generalize to unseen data.

The optimization analysis highlighted key trade-offs among the $4\,E$ indicators. Solar thermal energy was positively correlated with CO_2 mitigation, underlining the central role of collector efficiency for environmental performance. Likewise, increases in thermal energy translated into higher NPV, confirming that technical improvements also reinforce economic viability.

PSO performance was stabilized with constriction, inertia weight decay, and velocity clamping, avoiding swarm explosion and premature convergence. Multiple runs confirm consistent convergence, with robust Pareto fronts across climates. Semi-arid regions showed the most favorable conditions, maximizing NPV and CCM. These findings emphasize the value of reducing irreversibility and adapting configurations to specific climates.

5. Conclusions

The integration of thermoenergetic analysis, ANN surrogate modeling, and PSO–TOPSIS optimization proved effective in enhancing the performance of the hybrid GAX refrigeration system under different Mexican climates. The ANN achieved high predictive accuracy, supported by strong r values, low $RMSE$ and MAE, and statistically significant results, confirming its suitability for representing complex input–output relationships.

The optimization framework identified configurations that balanced technical, economic, environmental, and exergy criteria. Collector efficiency was decisive in maximizing both CO_2 mitigation and economic viability, with semi-arid climates offering particularly favorable outcomes. The PSO configuration, validated through convergence and robustness analyses, ensured stable and reliable results.

Overall, the study demonstrated that context-specific hybrid refrigeration systems provide scalable and sustainable cooling solutions. The methodology offers a replicable framework for integrating renewable energy into thermal systems and advancing sustainable energy applications under diverse climatic conditions.

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