On model parameter estimation methods of DC electric motors

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Summary

This paper describes the implementation of some parameter estimation methods commonly used for direct current (DC) electric motors. Model parameter estimation algorithms based on numerical optimization available in Simulink® are implemented to estimate the motor parameters. In addition, the algebraic identification approach is also applied for on-line parameter estimation. Some numerical comparisons of the presented estimation methods are included to show their satisfactory applicability scope.

Keywords: algebraic identification, DC electric motor, numerical optimization, parameter estimation

1. Introduction

Electric motors are used in diverse electric and mechatronic products and systems. Some applications of these electric actuators can be found in tool machines, robots, electric vehicles and hard disk drives. However, most of the motion or torque controllers require knowledge of some system parameters to get high energetic performance levels [1–4]. In this way, several important methodologies for system identification have been proposed (see, e.g., [5–7] and references therein). Most of the identification algorithms are mainly off-line, asymptotic and recursive. In fact, successful model parameter estimation techniques have been integrated in commercial computational platforms as MATLAB® and Simulink®. Thus, the unknown parameters can be easily estimated by implementation of those computational tools through input-output measurements. Nevertheless, system parameters can change during the life cycle of the product. Additionally, some engineering applications could demand information on system parameters in real time for adaptive-like control implementation or fault monitoring. In this regard, in [8] an algebraic parametric identification framework has been recently proposed for on-line estimation scenarios. In fact, on-line algebraic parameter identification has been successfully applied in active vibration control of vibrating mechanical systems and balancing of rotating machinery [9–12].

This paper describes the implementation of some parameter estimation methods commonly used for direct current (DC) electric motors. Model parameter estimation algorithms based on numerical optimization available in Simulink® platform are implemented to identify the motor parameters. Specifically, optimization algorithms of gradient descent and nonlinear least squares for parameter estimation are described. Nevertheless, other optimization techniques can be also applied to electric machine, which are excluded in this paper, due to limitations of space. In addition, the algebraic identification approach is also applied for estimation of the input-output system parameters. Some numerical comparisons of the presented estimation methods are included to show their applicability scope. Thus, the presented study provides an useful insight of recent techniques for model parameter estimation of DC motors. Moreover, the described estimation methods can be implemented and extended by the interested reader for motion control, fault monitoring and energetic efficiency characterization of DC motors.

2. Mathematical model

Consider the mathematical model of a DC permanent magnet motor (1). The parameters of the electric subsystem are inductance L, resistance of the armature circuit R, back electromotive force constant k_e and motor torque constant k_m. Inertia moment J and viscous damping b are the parameters of the mechanical subsystem. In addition, $y = \omega$ is the angular velocity of the output shaft, i is the electric current, u is the control voltage applied to the input terminals of the armature circuit, and τ_L denotes the unknown constant load torque.

$$\begin{aligned} L\frac{di}{dt} &= -Ri - k_e\omega + u \\ J\dot{\omega} &= -b\omega + k_m i - \tau_L \\ y &= \omega \end{aligned} \tag{1}$$

DC electric motor (1) exhibits the structural property of differential flatness with flat output $y = \omega$ [13-15]. Hence, the system variables can be expressed in terms of differential functions of y as

$$\begin{aligned} y &= \omega \\ i &= \frac{J}{k_m}\dot{y} + \frac{b}{k_m}y + \frac{1}{k_m}\tau_L \\ u &= \frac{LJ}{k_m}\ddot{y} + \left(\frac{bL + RJ}{k_m}\right)\dot{y} + \left(\frac{k_m k_e + Rb}{k_m}\right)y + \frac{R}{k_m}\tau_L \end{aligned} \tag{2}$$

Therefore, the input-output system dynamics is given by

$$\ddot{y} + a_1\dot{y} + a_0 y = bu - P \tag{3}$$

with

$$a_0 = \frac{k_m k_e + Rb}{JL}, \quad a_1 = \frac{Lb + RJ}{JL}, \quad b = \frac{k_m}{JL}, \quad P = \frac{R}{JL}\tau_L \tag{4}$$

Note that one can also obtain the input-output representation (3) by using the transfer function of the system, including the influence of the load torque. The simulations using numerical optimization an algebraic identification were performed using the input-output model shown in (3).

3. Parameter estimation using numerical optimization

Equation (3) shows that the DC motor can be represented using a linear time-invariant second order, single-input, single-output (SISO) model. From a systems perspective, the output can be represented by (5) when additive disturbances are ignored

$$y(t) = G(q)u(t) \tag{5}$$

where $G(q)$ is the transfer function of the system [6]. When the model of a system is unknown it becomes necessary to apply system identification techniques to sets of input and output data to determine a model that fits the system's behavior. For the purposes of this paper, the DC motor model is known; however, equation (3) becomes a set of models when the parameters or coefficients are unknown. Let us denote these parameters using a vector θ. Then, to determine the most accurate model, these coefficients need to be determined or estimated.

To estimate parameters we can use several approaches and algorithms: prediction-error minimization (gradient descent, least squares), correlation (maximum likelihood criterion, pattern search), subspace methods (stochastic state estimation), etc. [6].The most widely used method is the least-squares minimization [6, 16].

3.1. Least-squares method

Consider the parameter vector of (3) $\theta = [a_1 \;\; a_0 \;\; b]^T$, made up with the transfer function coefficients. We can express (3) with θ to be determinedas [16]:

$$[\dot{y} \quad y \quad -u]\begin{bmatrix} a_1 \\ a_0 \\ b \end{bmatrix} = -\ddot{y} \tag{6}$$

The row vector at the beginning of (6) is called the regression vector and denoted by φ. The right side of (6) will be the predictor or observed variable $\mathcal{y}$. Since y and u are time dependent, the time derivatives must be estimated using numerical methods (forward-Euler, back-Euler, etc.). For n corresponding values of $\mathcal{y}$ and φ we will have a n-size set of model data, which can be expressed as

$$Y = \Phi\theta \tag{7}$$

Where $Y = [\mathcal{y}_1 \;\; \mathcal{y}_2 \dots \mathcal{y}_n]^T$ and $\Phi = [\varphi_1 \;\; \varphi_2 \dots \varphi_n]^T$, assuming no noise. The least-squares criterion, with prediction error $\varepsilon = Y - \Phi\theta$, using a quadratic norm $\frac{1}{2}\varepsilon^2$, is given by [6]

$$V = [Y - \Phi\theta]^T[Y - \Phi\theta] \tag{8}$$

Then, the estimated parameter vector $\hat{\theta}$ will be given by the minimizing argument of (8) [6]:

$$\hat{\theta} = (\Phi^T\Phi)^{-1}\Phi^T Y \tag{9}$$

As it can be seen from (6) and (7), the output and its first and second derivative must be known *a priori*, and a number of measured data is necessary to perform the estimation, making the least-squares estimate an iterative process.

3.2. Gradient descent method

This method also consists in minimizing an error between a target output and a measured output. The algorithms consist in computing a difference between the desired output and the measured output

$$\varepsilon = y - \hat{y} \tag{10}$$

Where the measured output is $\hat{y} = \varphi\hat{\theta}$. The same criterion will be used as in the least-squares method, with a quadratic norm $\frac{1}{2}\varepsilon^2$. This method is based upon the fact that a function depending on a variable, in this case θ, decreases more steeply in the direction opposite of the function's gradient. Thus, the parameter vector is estimated through

$$\hat{\theta}_{i+1} = \theta_i - \nabla\varepsilon \tag{10}$$

Again, one can observe the iterative nature of this method. In fact, this method is usually much less efficient than the least squares.

3.3. Parameter estimation using MATLAB®

The MathWorks®, Inc. provides the package Simulink® Design Optimization™ (SDO), which can perform parameter estimation tasks through a Simulink® block model, a graphical user interface (GUI) and using numerical optimization. SDO performs the estimations using different methods, including non-linear least squares, gradient descent, pattern search and simplex search, and different algorithms, like Levenberg-Marquardt, trust region reflective, interior point, etc. The main window of the GUI is shown in Figure 1.The process to perform this tasks is listed as follows:

- Selecting "Parameter Estimation" in the model's window. The GUI will open with a default "Estimation Task" tab open.
- In the "Transient Data" tab the input-output measured data must be specified, as shown in Figure 2. For the purposes of this paper the data was imported from

simulations of a DC machine of known parameters. The model can consider several inputs and outputs and their respective measured values.

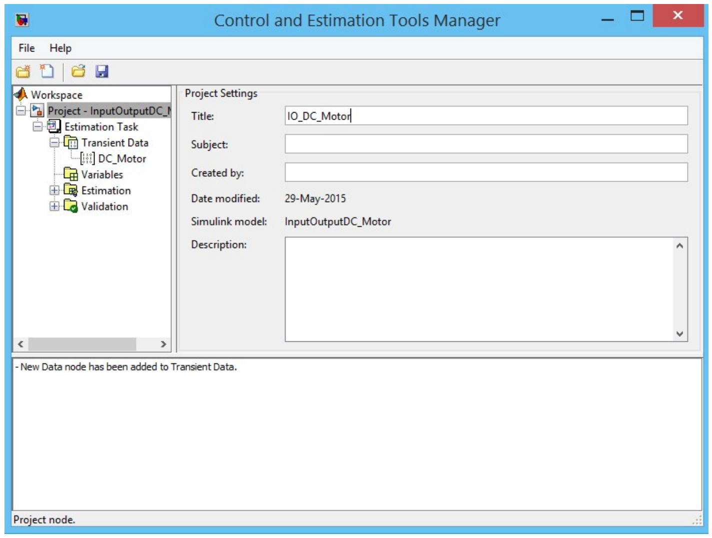


Fig. 1. GUI for the parameter estimation using Simulink®.

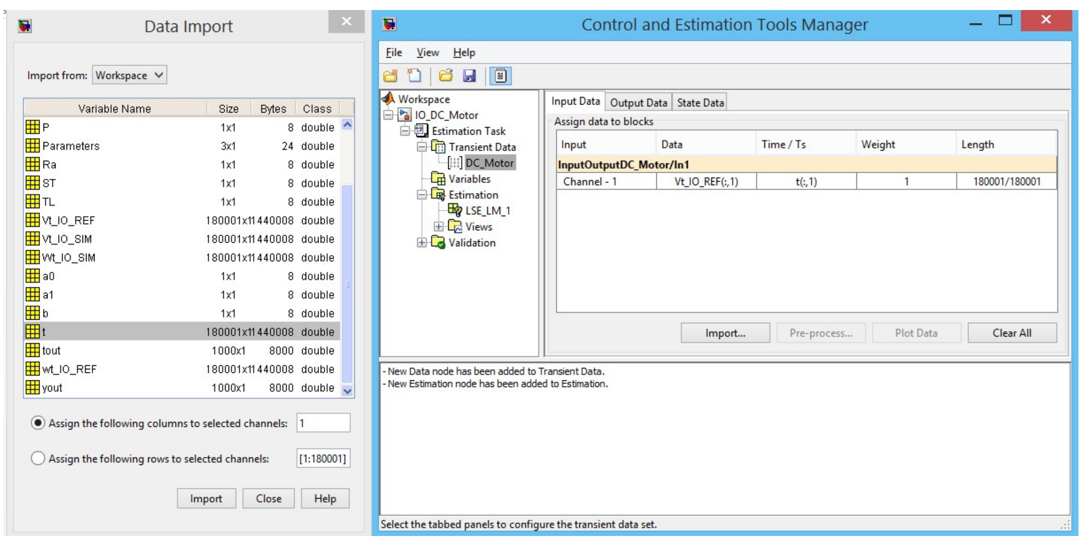


Fig. 2. Transient Data Tab of the GUI.

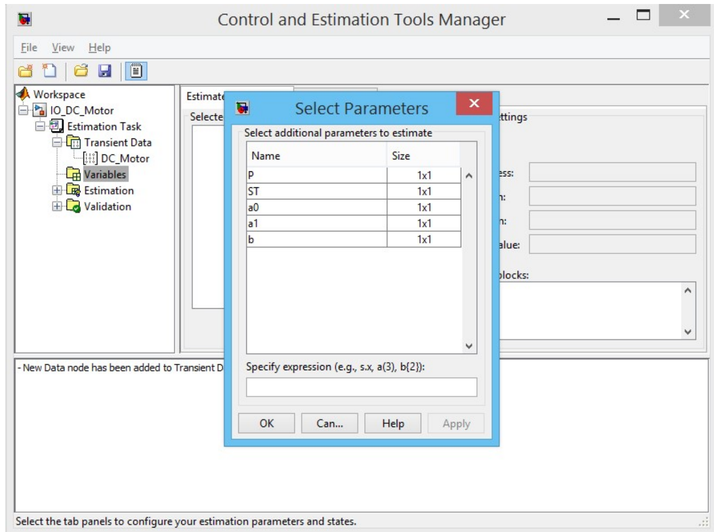


Fig. 3. Variables Tab of the GUI.

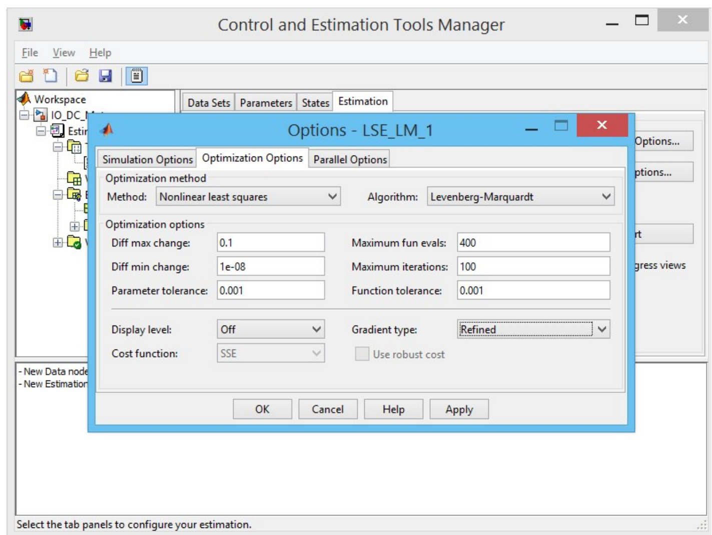


Fig. 4. Estimation Options Tab of the GUI.

- In the "Variables" tab (Figure 3) the parameters to be determined are selected. There is the option of estimating all the parameters of the model or just the ones that are required. This is particularly advantageous when some of the parameters are already known. For each parameter there is convenient to set its maximum and minimum value, as well as its typical value.

- In the “Estimations” tab several configuration options need to be set. First, we set the data sets to be used for estimation, which were previously loaded in the “Transient Data” tab. Then the “Parameters” tab gives the option to select the parameters to be estimated; again, we can use this option to perform different estimation tasks for the same parameters excluding some of them. The “States” tab shows the different states of the model, given by the integrator blocks. Finally in the “Estimation” there is an “Estimation Options” button that opens a window where we can set the simulation configuration, which should match that of the Simulink® model, the optimization options, where we choose the method to estimate the parameters, the algorithm to be used and some tolerance values depending on the desired accuracy. Figure 4 shows the window where these options are set.
- When the configurations are set, pressing “Start” initiates the estimation task. During the estimation plots of the parameter trajectories and the respective measured and simulated data are shown, which change with every iteration.

For further information, the reader should turn to Simulink® Design Optimization™ Documentation [17].

4. Algebraic parameter estimation

Consider the input-output model (3) where measurements of the output velocity y and the voltage input u are available for on-line estimation of parameters a_0, a_1 and b. First of all, system (3) is described in notation of Mikusiński operational calculus [18, 19] as follows

$$s^2Y(s) - sy_0 - \dot{y}_0 + a_1[sY(s) - y_0] - a_0Y(s) = bU(s) - \frac{P}{s} \tag{11}$$

Where $y_0 = y(t_0)$ and $\dot{y}_0 = \dot{y}(t_0)$ are unknown constants denoting the system initial conditions at time $t_0 \geq 0$.

Multiplying equation (11) by s, we have

$$s^3Y(s) - s^2y_0 - s\dot{y}_0 + a_1[sY(s) - y_0] - a_0Y(s) = bU(s) - \frac{P}{s} \tag{12}$$

To eliminate the influence of the initial conditions and load torque, equation (12) is differentiated three times with respect to the complex variable s, resulting in

$$\begin{aligned} &6Y(s) + 18s\frac{dY(s)}{ds} + 9s^2\frac{d^2Y(s)}{ds^2} + s^3\frac{d^3Y(s)}{ds^3} \\ &+a_1\left[6\frac{dY(s)}{ds} + 6s\frac{d^2Y(s)}{ds^2} + s^2\frac{d^3Y(s)}{ds^3}\right] \\ &+a_0\left[3\frac{d^2Y(s)}{ds^2} + s\frac{d^3Y(s)}{ds^3}\right] = b\left[3\frac{d^2U(s)}{ds^2} + s\frac{d^3U(s)}{ds^3}\right] \end{aligned} \tag{13}$$

Next, to avoid differentiation with respect to time, equation (13) is multiplied by s^{-3}

$$\begin{aligned} &6s^{-3}Y(s) + 18s^{-2}\frac{dY(s)}{ds} + 9s^{-1}\frac{d^2Y(s)}{ds^2} + \frac{d^3Y(s)}{ds^3} \\ &+a_1\left[6s^{-3}\frac{dY(s)}{ds} + 6s^{-2}\frac{d^2Y(s)}{ds^2} + s^{-1}\frac{d^3Y(s)}{ds^3}\right] \\ &+a_0\left[3s^{-3}\frac{d^2Y(s)}{ds^2} + s^{-2}\frac{d^3Y(s)}{ds^3}\right] = b\left[3s^{-3}\frac{d^2U(s)}{ds^2} + s^{-2}\frac{d^3U(s)}{ds^3}\right] \end{aligned} \tag{14}$$

Equation (14) is then transformed back to the time domain by applying operational calculus rules [18, 19], by associating $\frac{d^\nu}{ds^\nu}$, $\nu \geq 0$, with $(-1)^\nu t^\nu$:

$$\begin{aligned} &6\int^{(3)}y - 18\int^{(2)}ty + 9\int t^2y - t^3y \\ &+a_1\left[6\int^{(3)}ty + 6\int^{(2)}t^2y - \int t^3y\right] \\ &+a_0\left[3\int^{(3)}t^2y - \int^{(2)}t^3y\right] = b\left[3\int^{(3)}t^2u - \int^{(2)}t^3u\right] \end{aligned} \tag{15}$$

For convenience in notation, integral $\int_0^t \varphi(\tau)\, d\tau$ is described as $\int \varphi$ and integral $\int_0^t \int_0^\tau \varphi(\rho) d\rho d\tau$ as $\int^{(2)} \varphi$ and so on.

Equation (15), after some more integrations, leads to the linear system of equations

$$A(t)\theta = B(t) \tag{16}$$

where

$$\theta = \begin{bmatrix} a_0 \\ a_1 \\ b \end{bmatrix}, A(t) = \begin{bmatrix} a_{11}(t) & a_{12}(t) & a_{13}(t) \\ a_{21}(t) & a_{21}(t) & a_{23}(t) \\ a_{31}(t) & a_{32}(t) & a_{33}(t) \end{bmatrix}, B(t) = \begin{bmatrix} b_1(t) \\ b_2(t) \\ b_3(t) \end{bmatrix}$$

with

$$a_{11} = 3\int^{(3)} t^2 y - \int^{(2)} t^3 y$$

$$a_{12} = -6\int^{(3)} ty + 6\int^{(2)} t^2 y - \int t^3 y$$

$$a_{13} = \int^{(2)} t^3 u - 3\int^{(3)} t^2 u$$

$$b_1 = -6\int^{(3)} y + 18\int^{(2)} ty - 9\int t^2 y + t^3 y$$

$$a_{21} = \int a_{11}, \;\; a_{31} = \int a_{21}$$

$$a_{22} = \int a_{12}, \;\; a_{32} = \int a_{22}$$

$$a_{23} = \int a_{13}, \;\; a_{33} = \int a_{23}$$

$$b_2 = \int b_1, \;\; b_3 = \int b_2$$

Hence, from (16), parameter vector is given by

$$\theta = A^{-1}B \tag{17}$$

Then,

$$a_0 = \frac{\Delta_1}{\Delta}$$
$$a_1 = \frac{\Delta_2}{\Delta} \quad (18)$$
$$b = \frac{\Delta_3}{\Delta}$$

Nevertheless, a possible inconvenience is the presence of singularities when some $\Delta_1 \equiv 0, i = 1, \dots, 3$. To avoid this situation and considering the fact that system parameters have to be positive constants, the algebraic identifiers are obtained as

$$\hat{a}_0 = \frac{\int |\Delta_1|}{\int |\Delta|}$$
$$\hat{a}_1 = \frac{\int |\Delta_2|}{\int |\Delta|} \quad (19)$$
$$\hat{b} = \frac{\int |\Delta_3|}{\int |\Delta|}$$

5. Simulation results

Some computer simulations were performed to verify the effectiveness of the algebraic identification and estimation algorithms based on numerical optimization for a DC motor characterized by the set of parameters described in Table 1.

Given the parameters shown in Table1, the input-output model described by (3) was built. Therefore, the parameter vector to be identified with estimation methods is $\theta = [1895.36 \quad 64.03 \quad 110849.06]^T$.

$$R = 7\ \Omega$$

$$L = 120\ \mathrm{mH}$$

$$k_m = 14.1\ \mathrm{mN \cdot m/A}$$

$$k_e = 14.1\ \mathrm{mV \cdot s/rad}$$

$$J = 1.06 \times 10^{-6} \mathrm{kg \cdot m^2}$$

$$b_m = 6.04 \times 10^{-6} \mathrm{N \cdot m \cdot s}$$

Table 1. Parameters of the DC electric motor.

5.1. Parameter estimation using numerical optimization

Several simulations of the DC motor were performed using Simulink® DesignOptimization™. Estimation was performed using the methods and algorithms available, however, only the results obtained by using the approaches of the non-linear least-squares (LS) and the gradient descent (GD) are presented in detail due to space limitations and because these methods manifested better performance.

Table 1 shows the parameters of the DC motor analyzed. The speed output and the input voltage were used for the parameter estimation process. The parameters were initially varied 5% of their known values. Table 2 shows the real parameters and the parameters used for numerical simulations. With each iteration, the parameters were tuned and used in the next iteration. After the estimation using the different algorithms, the difference between the initial values proposed and the estimated values was 0% in most cases, meaning that the parameters were estimated correctly.

Parameter	Real Value	Initial Value	Initial error %
$a_0[\mathrm{s}^{-2}]$	1895.3616	1800.5936	5.00
$a_1[\mathrm{s}^{-1}]$	64.0314	67.2330	5.00
$b\ [\mathrm{s}^{-3}\mathrm{V}^{-1}]$	110849.0566	105306.6038	5.00

Table 2. Parameters used during the simulations.

The results presented in this paper were obtained using the least-squares method and the gradient descent method with all the available algorithms in the SDO. For the least-squares method, a trust-region-reflective (TRR) and a Levenberg-Marquardt (LM) algorithm were used. For the gradient descent method, an active-set (AS), interior-point (IP), sequential quadratic programming (SQP) algorithms were used, along with a TRR algorithm as well. Most of the simulations resulted in the estimated parameters fitting the real ones, with the exception of the SQP. Table 3 shows, however, the number of iterations that every algorithm needed to perform the estimation.

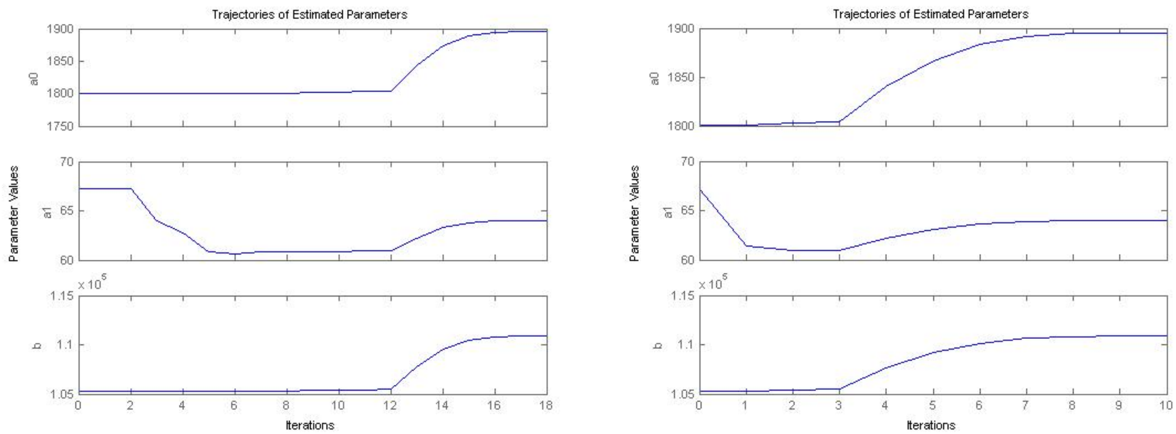
Method	Algorithm	Number of Iterations	Final Error %
LS	TRR	10	0.00
	LM	10	0.00
GD	AS	32	0.00
	IP	36	0.00
	TRR	18	0.00
	SQP	13	4.94

Table 3. Result of the estimation using numerical optimization.

As one can see from these results, every method required at least 10 iterations and all the parameters were estimated correctly with the exception of the SQP algorithm. Figure 5 shows the typical parameter trajectory plots obtained from the GUI. Because of the space limitations, only the LM and the TRR plots are shown. Hence the described numerical optimization algorithms available in Simulink® represent an alternative solution for off-line estimation of the motor parameters and next the estimates can be used for implementation of some control scheme.

5.2. Parameter estimation using algebraic identification

On the other hand, Figs. 6(a)-6(c) describe the on-line estimation of the system parameters using the algebraic identifiers (19). Clearly, one can observe the effectiveness of the algebraic identification method for the fast estimation of the parameters of the input-output system model (3) before 0.5 s. Therefore, on-line algebraic identification qualifies as good choice to be implemented with some adaptive-like control scheme.



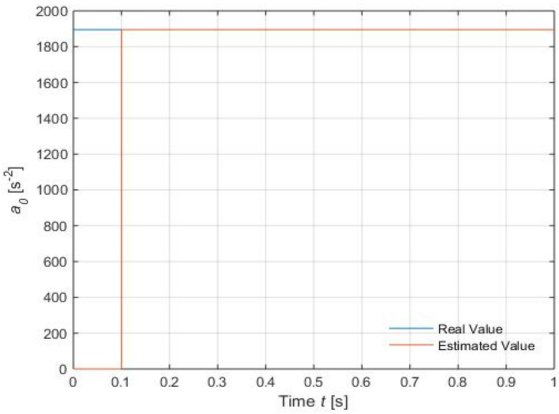
a) Gradient Descent Method, Trust-Region-Reflective Algorithm

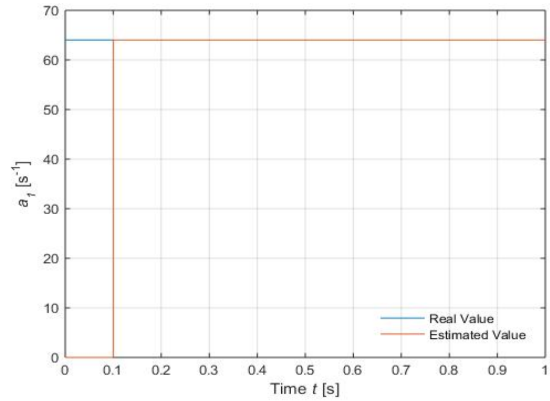
b) Least-Squares Method, Levenberg-Marquardt Algorithm

Fig. 5. Parameter Trajectory plots for the estimated parameters.

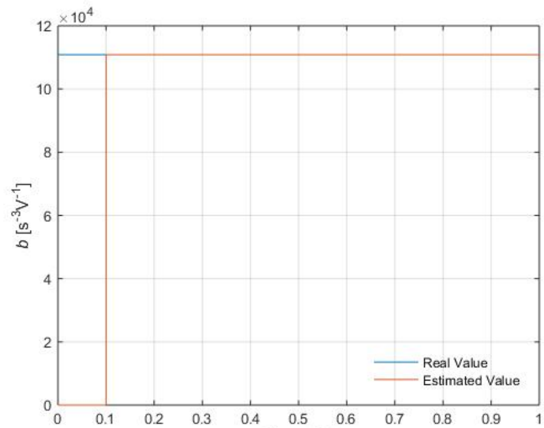
6. Conclusions

In this paper, several parametric identification methods have been described for on-line and off-line estimation of the parameters associated with a DC motor. Recent estimation algorithms based on numerical optimization available in Simulink® platform were presented yielding to satisfactory results for off-line parameter estimation tasks. On the other hand, algebraic identifiers were synthesized for on-line parameter estimation. Numerical comparisons of the presented estimation methods were included, showing their satisfactory applicability scope.





a) Algebraic Estimation of parameter a_0.



b) Algebraic Estimation of parameter a_1.

c) Algebraic Estimation of parameter b.

Fig. 6. Parameter Trajectory plots for the estimated parameters using algebraic method.

7. References

[1] S. F. Alyaqout, P. Y. Papalambros, A. G. Ulsoy, "Combined Robust Design and Robust Control of an Electric DC Motor". Mechatronics IEEE/ASME. Vol. 16. 2011. 574-582 pp.

[2] Z. Li, J. Chen, G. Zhang, M. G. Gan, "Adaptive robust control for DC motors with input saturation, Control Theory & Applications". IET. Vol. 5. 2011. 1895-1905 pp.

[3] C. F. Caruntu, C. Lazar, "Robustly stabilizing model predictive control design for networked control systems with an application to direct current motors, Control Theory & Applications". IET. Vol. 6. 2012. 943-952 pp.

[4] G. Murtaza, A. I. Bhatti, Control of DC motors using sliding mode, Applied Sciences and Technology (IBCAST). 9th International Bhurban Conference, 2012. 37-42 pp.

[5] Identification of Dynamic Systems, Springer-Verlag. Berlin. 2011.

[6] L. Ljung, Systems Identification: Theory for the User. 1987. Prentice-Hall. Upper Saddle River, NJ.

[7] T. Soderstrom, P. Stoica, System Identification. 1989. Prentice-Hall. New York.

[8] M. Fliess, H. Sira-Ramírez, "An algebraic framework for linear identification". ESAIM: Control. Optimization and Calculus of Variations. Vol. 9. 2003. 151-168 pp.

[9] F. Beltran-Carbajal, G. Silva-Navarro, "On the algebraic parameter identification of vibrating mechanical systems". International Journal of Mechanical Sciences. Vol. 92. 2015. 178-186 pp.

[10] M. Arias-Montiel, F. Beltran-Carbajal, G. Silva-Navarro, "On-line algebraic identification of eccentricity parameters in active rotor-bearing systems". International Journal of Mechanical Sciences. Vol. 85. 2014. 152-159 pp.

[11] F. Beltran-Carbajal, G. Silva-Navarro, "Adaptive-like vibration control in mechanical systems with unknown parameters and signals" Asian Journal of Control. Vol. 16. No. 168. 2013. 1613-1626 pp.

[12] F. Beltran-Carbajal, G. Silva-Navarro, M. Arias-Montiel, "Active unbalance control of rotor systems using on-line algebraic identification methods". Asian Journal of Control. Vol. 16. No. 16. 2013. 1627-1637 pp.

[13] F. Beltran-Carbajal, A. Favela-Contreras, A. Valderrabano-Gonzalez, J. C. Rosas-Caro, "Output feedback control for robust tracking of position trajectories for DC electric motors". Electric Power Systems Research. No. 107. 2014. 183-189 pp.

[14] F. Beltran-Carbajal, A. Valderrabano-Gonzalez, J. C. Rosas-Caro, A. Favela-Contreras, "An asymptotic differentiation approach of signals in velocity tracking control of DC motors". Electric Power Systems Research. No. 122. 2015. 218-223 pp.

[15] M. Fliess, J. Lévine, P. Martin, P. Rouchon, "Flatness and defect of non-linear systems: introductory theory and examples". International Journal of Control. Vol. 61. 1995. 1327-1361 pp.

[16] S. S. Saab, R. A. Kaed-Bey, "Parameter identification of a DC motor: an experimental approach, Electronics, Circuits and Systems". ICECS. The 8th IEEE International Conference. Vol. 2. No. 1. 2001. 981,984 pp.

[17] Simulink® Design Optimization ™. TheMathWorks, Inc. 2014.

[18] J. Mikusiński, Operational Calculus, PWN & Pergamon. 2nd Ed. Vol. 1. 1983. Warsaw, Poland.

[19] H. J. Glaetske, A. P. Prudnikov, K. A. Skrnik, Operational Calculus and Related Topics. 2006. Chapman and Hall/ CR. Boca Raton, FL.