

TRAJECTORY ANALYSIS FOR THE DIFFERENT STRATEGIES OF CIRCUIT DESIGN

ANÁLISIS DE TRAYECTORIA PARA LAS DIFERENTES ESTRATEGIAS DE DISEÑO DE CIRCUITOS

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Abstract

Some design trajectories were analyzed using a generalized design methodology. The starting point of the design process was changed to obtain different trajectories and compare them by processor time. Study of the phase portrait, consisting of a family of trajectories, allows us to analyze the acceleration effect. A special line, called a separatrix, divides a set of trajectories into two parts: with and without possible acceleration. The numerical results of the design of passive and active electronic circuits prove that the optimal choice of the starting point of the design algorithm allows you to minimize the time of the design process.

Keywords: Control theory application, optimal start point selection, time-optimal design algorithm.

Resumen

Varias trayectorias de diseño se analizaron utilizando una metodología de diseño generalizada. El punto de partida del proceso de diseño se cambió para obtener diferentes trayectorias y compararlas por tiempo de procesador. El estudio del

retrato de fase, que consiste en una familia de trayectorias, nos permite analizar el efecto de aceleración. Una línea especial, llamada separatrix, divide un conjunto de trayectorias en dos partes: con y sin aceleración posible. Los resultados numéricos del diseño de circuitos electrónicos pasivos y activos demuestran que la elección óptima del punto de partida del algoritmo de diseño le permite minimizar el tiempo del proceso de diseño.

Palabras clave: *Aplicación de la teoría de control, algoritmo de diseño de tiempo óptimo, selección óptima del punto de inicio.*

1. Introduction

The problem of reducing computer time when designing large systems is one of the significant problems in the overall improvement of design quality. Some works devoted to this problem focus on how to reduce the number of operations when solving two main problems: circuit analysis and numerical optimization. By solving these problems successfully, one can reduce the total time required for analog circuit optimization and this fact serves as a basis for improving design quality.

The methods used to analyze complex systems are being improved continuously. Some methods reduce the time need for circuit analysis. This includes the well-known idea of using sparse matrix methods [Osterby, 1983], [George, 1984] and decomposition methods [Rabat, 1985]. Some alternative methods such as homotopy methods [Tadeusiewicz, 2013] were successfully applied.

The techniques for analog circuit optimization can be classified in two main groups: deterministic optimization algorithms and stochastic search algorithms. Practical methods of optimization were developed for circuit designing, timing, and area optimization [Brayton, 1981], [Ruehli, 1987]. However, classical deterministic optimization algorithms may have a number of drawbacks: they may require that a good initial point be selected in the parameter space, they may reach an unsatisfactory local minimum, and they require that the cost function be continuous and differentiable. To overcome these issues, special methods were applied to determine the initial point of the process by centering [Stehr, 2003] or applying geometric programming methods [Hershenson, 2001].

Stochastic search algorithms, especially evolutionary computation algorithms like genetic algorithms, differential evolution, genetic programming, particle swarm optimization, etc. have been developed in recent years [Nam, 2001], [Paulino, 2001], [Alpaydin, 2003], [Srivastava, 2007], [Liu, 2009], [Yengui, 2012], [Ridzuan, 2016], [Passos, 2017], [Venturelli, 2017], [Li, 2019], [Zadeh, 2019]. Genetic algorithms have been employed as optimization routines for analog circuits due to the ability to find a satisfactory solution. A special algorithm defined as a particle swarm optimization technique is one of the evolutionary algorithms and competes with genetic algorithms. This method is successfully used for electromagnetic problems and for optimization of microwave systems [Robinson, 2004], [Zaman, 2011].

A more general formulation of the circuit optimization problem was developed on a heuristic level some decades ago [Kashirskiy, 1979]. This approach ignored Kirchhoff's laws for all or part of a circuit during the optimization process. The practical aspects of this idea were developed for the optimization of microwave circuits [Rizzoli, 1990] and for the synthesis of high-performance analog circuits [Ochotta, 1996] in an extreme case where all the equations of the circuit were not solved during the optimization process.

On the other hand, a different approach based on optimal control theory can be used to minimize the number of operations and processor time. A generalized theory of system design based on the formulation of control theory was developed in some previous papers [Zemliak, 1999], [Zemliak, 2004], [Zemliak, 2014]. Thus, the process of circuit optimization was generalized and defined as the dynamic controllable system. In this case, the basic element is the control vector that changes the structure of the equations of the system of optimization process. Thus, there is a set of strategies of optimization that have different number of operations and different CPU times [Zemliak, 2016].

This approach serves to determine the time-optimal design algorithm. On the other hand, this approach makes it possible to analyze the design process with great clarity when moving along a trajectory in the design space. The basic concept of the theory is the introduction of special control functions, which, on the one hand, generalize the design process, and on the other hand, make it possible to control the

design process to achieve the optimal point of the objective design function in minimal computer time. This possibility appears because an almost infinite number of different design strategies exist within the framework of the theory, but different design strategies have different numbers of operations and computer runtimes. Within this concept, a traditional design strategy is just one representative of a huge set of different design strategies. As shown in [Zemliak, 2007], the potential gain in computer time, which can be obtained using the new formulation of the design problem, increases with increasing size and complexity of the system, but it is realized only if the optimal design trajectory is constructed. The main goal of the paper is to find opportunities to reduce CPU time in circuit design based on trajectory analysis of various design strategies. We can define the problem of searching for properties of an optimal designing trajectory as one of the first tasks that need to be solved in order to build an optimal algorithm.

2. Methods

The design process for any analog system design can be defined [Zemliak, 2004] as the problem of the generalized objective function $F(X, U)$ minimization by means of the vector equation 1.

$$X^{s+1} = X^s + t_s \cdot H^s \quad (1)$$

With the constraints equation 2.

$$(1 - u_j) \cdot g_j(X) = 0, j = 1, 2, \dots, M \quad (2)$$

Where $X \in R^N$, $X = (X', X'')$, $X' \in R^K$ is the vector of the independent variables and the vector $X'' \in R^M$ is the vector of dependent variables ($N = K + M$), $g_j(X)$ for all j is the system model, s is the iterations number, t_s is the iteration parameter, $t_s \in R^1$, $H \equiv H(X, U)$, is the direction of the generalized objective function $F(X, U)$ decreasing, U is the vector of the special control functions $U = (u_1, u_2, \dots, u_M)$, where $u_j \in \Omega$; $\Omega = \{0; 1\}$. The generalized objective function $F(X, U)$ is defined as: $F(X, U) = C(X) + \psi(X, U)$ where $C(X)$ is the ordinary design process cost function, and $\psi(X, U)$ is the additional penalty function: $\psi(X, U) = \frac{1}{\varepsilon} \sum_{j=1}^M u_j \cdot g_j^2(X)$. This problem formulation

permits to redistribute the computer time expense between the problem (equation 2) solve and the optimization procedure (equation 1) for the function $F(X, U)$. The control vector U is the main tool for the redistribution process in this case. A virtually infinite number of different design strategies appear because the vector U depends on the current optimization step. The number of strategies of structural basis is 2^M . The task of finding the optimal design strategy is currently formulated as a typical task of minimizing the functional in control theory. The functional that needs to minimize is the total CPU time T of the design process. This functional depends directly on the operations number and more generally on the designing trajectory that has been realized. The main difficulty in determining this problem is the unknown optimal dependencies of all control functions u_j . This problem is central to this type of design process definition.

3. Results

The problem of the initial point selection for the design process is one of the essential problems of the time-optimal algorithm construction. Let's analyze the design process of the simplest electronic circuit in figure 1.

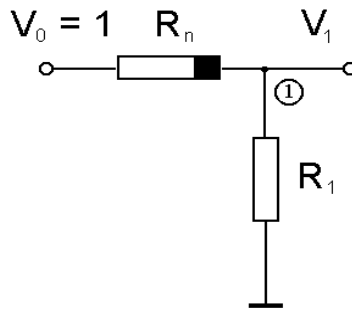


Figure 1 Simplest one node circuit.

The circuit has one node ($M = 1$). The vector of the state variables X has two components $X = (x_1, x_2)$ where $x_1^2 = R_1$, $x_2 = V_1$. The nonlinear element has the following dependency: $R_n = r_0 + bV_1$. Using the Laws of Kirchhoff we can obtain the following function $g(X)$ by equation 3.

$$g_1(X) \equiv x_2(x_1^2 + r_0 + bx_2) - x_1^2 = 0 \quad (3)$$

The objective function is defined by the formula $C(X) = (x_2 - k_V)^2$, where k_V has the fixed value. There is only one control function u_1 in this case because there is only one dependent parameter x_2 . The design trajectory for this example is the curve in two-dimensional space, if the numerical design algorithm is applied. The optimization procedure and the electronic system model, in accordance with the new design methodology [Zemliak, 1999], are defined by the next equations 4 and 5.

$$x_i^{s+1} = x_i^s + t_s \cdot f_i(X, U), \dots i = 1,2 \quad (4)$$

$$(1 - u_1) \cdot g_1(X) = 0 \quad (5)$$

Where U is the vector of control variables, and the components of the movement directions $f_i(X, U)$ for the $i = 1,2$ depend on the optimization method. These functions, for the gradient method for example, are given by the equation 6.

$$f_1(X, U) = -\frac{\delta}{\delta x_1} F(X, U)$$

$$f_2(X, U) = -u_1 \frac{\delta}{\delta x_2} F(X, U) + \frac{(1 - u_1)}{t_s} [-x_2^s + \eta_2(X)] \quad (6)$$

Where $F(X, U)$ is the generalized objective function, $F(X, U) = C(X) + \frac{1}{\varepsilon} u_1 \cdot g_1^2(X)$, $\eta_2(X)$ is the implicit function ($x_2^{s+1} = \eta_2(X)$) and it gives the value of the parameter x_2 from the equation 5, and the operator $\frac{\delta}{\delta x_i}$ for $i = 1,2$ means: $\frac{\delta}{\delta x_1} F = \frac{\partial F}{\partial x_1} + \frac{\partial F}{\partial x_2} \frac{\partial x_2}{\partial x_1}$, $\frac{\delta}{\delta x_2} F = \frac{\partial F}{\partial x_2}$. In this case, we have only two different strategies in a structural basis.

We named the first strategy with $u_1 = 0$ as Traditional Design Strategy (TDS), and the second strategy with $u_1 = 1$ as Modified Traditional Design Strategy (MTDS).

The family of the design curves for the circuit on figure 1, which corresponds to the MTDS, is shown in figure 2 for the 2-D phase space. These curves have different start points but the same final point F. Starting points were selected on a vertical line and have different initial coordinate for x_2 . The special curve S-F, which is marked by bold line, is the separating curve and can be named as separatrix.

This curve separates the trajectories that are the candidates for the acceleration effect achievement (all curves that lie under the curve S-F), and the trajectories that cannot produce the acceleration effect (curves that lie over the curve S-F). The

acceleration effect appears when the control vector switches to using different design strategy, and its computational time is many times reduced compared to other strategies. A vertical dashed line P crosses the end point, and all curves that have a chance to realize the acceleration effect. At least there is a probability of this effect under the initial conditions $x_2 = [-2, 0.5]$.

It is clear that the projections of the final point F to all curves of the first group define the switching point of the optimal trajectory, which produces the acceleration effect. The comparison of the number of iterations as the function of the initial coordinate x_2 is shown in figure 3.

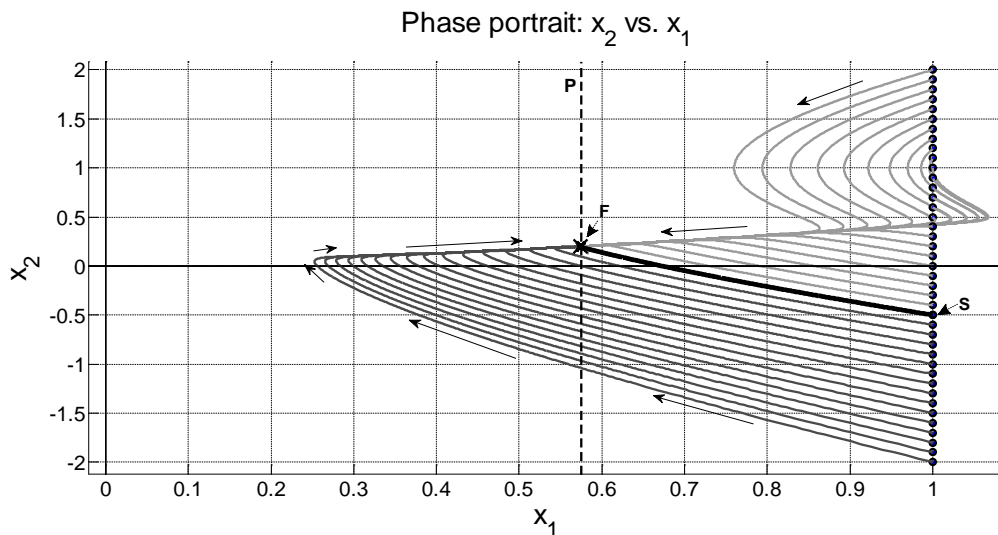


Figure 2 Trajectories of the MTDS for various start points.

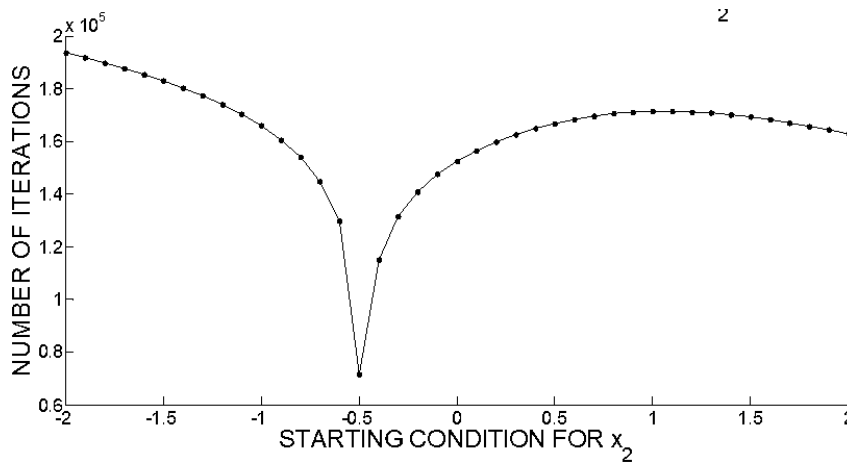


Figure 3 Number of iterations as the function of the initial coordinate x_2 .

We can see a strict dependence of the number of iterations on the initial value of the variable x_2 and on the trajectories in figure 2. The minimal point of the curve corresponds to the separatrix of the figure 2, but only the curves of this figure that lies under the separatrix i.e. that correspond to the initial value for $x_2 < -0.5$ can produce the acceleration effect.

The minimum point corresponds to the separatrix of figure 2, but only the curves of this figure that lie under the separatrix, i.e. correspond to the initial value of $x_2 < -0.5$ can create an acceleration effect. The results of the analysis of TDS, MTDS and the optimal strategy with a single switching point are shown in table 1.

Table 1 Data of TDS, MTDS and optimal strategy.

| Control vector | Iterations number | Total design time (ms) | Time Gain |
|----------------|-------------------|------------------------|-----------|
| (0) | 100419 | 8.034 | 1 |
| (1) | 71461 | 5.645 | 7.04 |
| (1)-(0) | 96 + 1 | 0.008 | 1048 |

Table 1 shows the number of iterations for TDS, MTDS and optimal strategy, as well as the total design time and time-gain relative to TDS. We can see that strategy (equation 1) has a time gain of about 7 times, but an optimal strategy with one switching point has a time gain of 1048 times with respect to TDS.

The N-dimensional case has been analyzed below. The second example corresponds to the two-node circuit in figure 4.

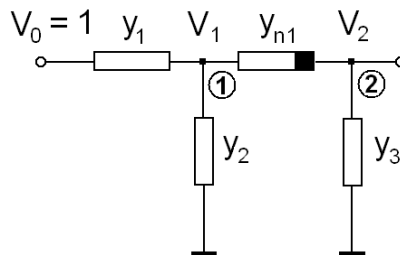


Figure 4 Two-node circuit topology.

The phase space of the total states parameters has five dimensions. The separate lines are transformed to the separate hyper-surfaces in this case. We can study the phase projections of the separate hyper-surfaces in this case.

This circuit has three independent variables as admittances y_1, y_2, y_3 ($K = 3$) and two dependent variables as nodal voltages V_1, V_2 ($M = 2$). The nonlinear element of the circuit has a dependence: $y_{n1} = a_n + b_n \cdot (V_1 - V_2)^2$. The nonlinearity parameter b_n is 1.0. The vector of state parameters X includes five components: $x_1^2 = y_1, x_2^2 = y_2, x_3^2 = y_3, x_4 = V_1, x_5 = V_2$. The optimization process system includes five equations, and the circuit model includes two equations. The functions $g_j(X)$ of system (Equation 2) are determined by the equation 7.

$$g_1(X) \equiv (1 - x_4)x_1^2 - (x_4 - x_5)(y_0 + a(x_4 - x_5)^2) - x_4x_2^2 = 0$$

$$g_2(X) \equiv (x_4 - x_5)(y_0 + a(x_4 - x_5)^2) - x_5x_3^2 = 0 \quad (7)$$

The objective function $C(X)$ has been determined by the formula $C(X) = (x_5 - m_1)^2$, where m_1 is a beforehand-defined output voltage of the divider. This circuit is characterized by two dependent parameters (two nodal voltages) and the control vector includes two control functions: $U = (u_1, u_2)$. The structural basis of the design strategies includes four design strategies with control vectors: (00), (01), (10), and (11).

The results of the analysis of the complete structural basis of various design strategies and some complex strategies for the circuit are shown in table 2 and figure 5 and figure 6.

Table 2 Data of complete structural basis of designing strategies.

| Control vector | Iterations number | Total design time (ms) | Time gain |
|----------------|-------------------|------------------------|-----------|
| (00) | 176837 | 381.680 | 1 |
| (01) | 23149 | 54.208 | 7.041 |
| (10) | 245297 | 549.691 | 0.694 |
| (11) | 99253 | 103.503 | 3.688 |
| (11) - (00) | 4 + 1 | 0.006 | 60300 |
| (01) - (00) | 6 + 1 | 0.016 | 23547 |

Table 2 shows the number of iterations for each strategy used, the total design time and the gain in computational time with respect to TDS. We can see that strategy (01) has the gain of computer time approximately 7 times with respect to TDS and the MTDS with control vector (11) has the CPU time gain 3.68 times.

The acceleration effect for strategies (11)→(00) and (01)→(00) shows the possibility of minimizing the CPU time many times by switching between different design strategies. In this case, for the complex strategy (11)→(00), the CPU time gain increases by approximately 60300 times.

To realize the effect of acceleration, we need to study the phase trajectories of MTDS and, possibly, other strategies for different starting points.

Phase trajectories lie in the space R^5 , and in this case we can plot the projections of spatial curves. More informative projections correspond to the plane $x_3 - x_5$. These projections for MTDS curves are shown in figure 5, and the curves shown in figure 6 were obtained using strategy (01).

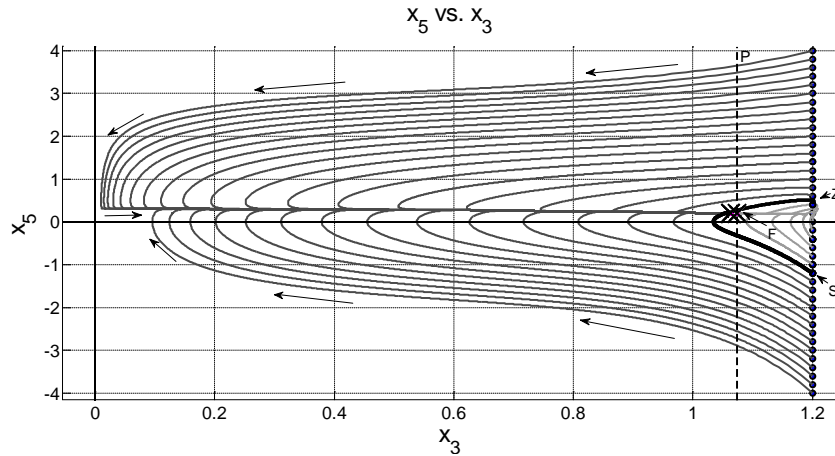


Figure 5 Projections $x_3 - x_5$ of phase diagrams for MTDS for a two-node circuit.

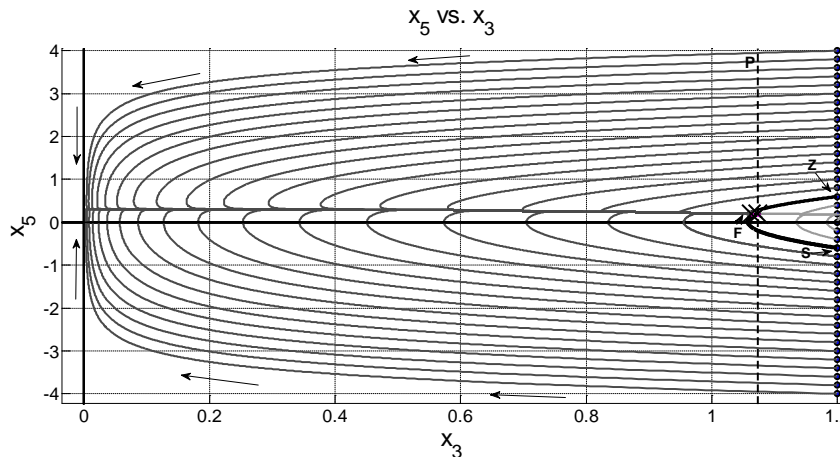


Figure 6 Projections $x_3 - x_5$ of phase diagrams for strategy (01) for a two-node circuit.

These families of curves were obtained for different starting points. The end point was marked with a cross (point F). P is the line that defines the possible projections of the switch point to the endpoint. The curve S-F-Z is the projection of the separatrix. Black lines are curves with the possibility of acceleration when jumping to an end point. Gray lines are curves that do not allow you to get an acceleration effect. The separatrix (bold black line) separates both zones with and without acceleration effect. The vertical line P intersects the end point and all the curves that have the possibility of realizing the acceleration effect.

The active nonlinear circuit is analyzed below. This is a single-stage transistor amplifier, shown in figure 7.

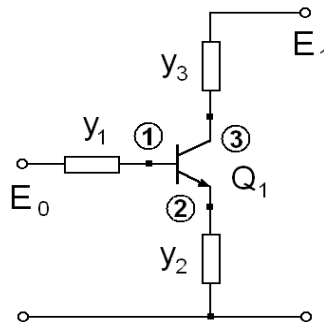


Figure 7 Circuit topology for one-stage transistor amplifier.

The Ebers-Moll static model of the transistor has been used [Massobrio, 1993]. We define three independent variables y_1, y_2, y_3 as admittance ($K = 3$) and three dependent variables V_1, V_2, V_3 as nodal voltages ($M = 3$). The control vector includes three components $U = (u_1, u_2, u_3)$. The structural basis of design strategies includes eight strategies: (000), (001), (010), (011), (100), (101), (110), (111).

Applying the Kirchhoff law for this circuit, we can write three equations 8.

$$\begin{aligned}
 g_1 &= I_B - (E_1 - V_B) \cdot y_1 = 0 \\
 g_2 &= I_E - V_E \cdot y_2 = 0 \\
 g_3 &= I_C - (E_2 - V_C) \cdot y_3 = 0
 \end{aligned}
 \tag{8}$$

The state parameter vector X includes six components: $x_1^2 = y_1, x_2^2 = y_2, x_3^2 = y_3, x_4 = V_1, x_5 = V_2, x_6 = V_3$. The optimization procedure includes 6 equations.

The system model is converted to the equation 9.

$$\begin{aligned}
 g_1(X) &= I_B - (E_1 - x_4) \cdot x_1^2 \\
 g_2(X) &= I_E - (x_2^2 \cdot x_5) \\
 g_3(X) &= I_C - (E_2 - x_6) \cdot x_3^2
 \end{aligned}
 \tag{9}$$

In this case, the phase trajectories lie in R^6 . Analyzing the various projections of phase curves, we can state that the projections x_3-x_6 are the most informative. We analyzed all structural basis strategies and complex strategies with one and two switching points. As a result of the analysis of various strategies and their projections, we obtained the following results:

- TDS (000) includes 2099235 steps and has CPU time $t = 297.6$ sec.
- MTDS (111) includes 6024 steps and has CPU time $t = 0.071$ sec.
- The complex strategy (111)-(000)-(111) with two switching points in step 49 (switch from strategy (111) to strategy (000)) and in step 50 (switch from strategy (000) to strategy (111)).

The projections of three principal strategies, TDS, MTDS and optimal complex strategy (111)-(000)-(111) are shown in figure 8.

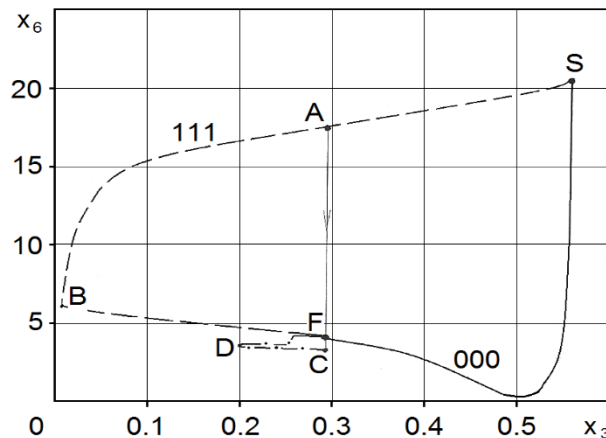


Figure 8 Projections $x_3 - x_5$ of phase diagrams for one-stage transistor amplifier.

In this case, we compare three different strategies: TDS (000), then MTDS (111) and the complex strategy (111) - (000) - (111) with two switching points. S is the starting point of the design process, F is the end point of the design process, A is the optimal position of the first switching point.

TDS corresponds to the S-F curve (solid line). MTDS corresponds to the S-A-B-F curve (dashed line). A complex strategy with two switching points corresponds to the S-A-C-D-F curve.

The total number of iterations of the optimal complex strategy is 1464, and the processor time $t = 0.018$ sec. The time gain in this case is 16533 times.

4. Discussion

We studied phase portraits of various design strategies when designing some circuits. This helps us to understand well the emergence of the possibility of significantly accelerating the design process. We see that it is possible to get an acceleration effect by applying a complex strategy consisting of MTDS and TDS strategies. It is very important that the MTDS curves cover the end point of the design process. We can create an acceleration effect by defining the switching point between MTDS and TDS at the optimal integration step. This optimal step corresponds to the intersection of the projection of the endpoint onto the MTDS curve. The concept of the separatrix is very fruitful for determining the optimal position of the starting point of the design process. It can be noted that the MTDS curves lying inside the space limited by separatrix cannot cause any acceleration. Only strategies outside the separatrix S are candidates for the acceleration effect. Implementation of a complex strategy consisting of TDS and MTDS with one or two switching points allows us to significantly speed up the design process. It can be stated that the analysis of various design strategies gives us the opportunity to realize the acceleration effect and get a time gain of 4-5 orders of magnitude.

5. Conclusions

The task of constructing a time-optimal algorithm can be solved as a problem of time minimization on the control theory. In this case, the overall computer time for system design can be significantly reduced. On the other hand, the problem of real construction of a time-optimal algorithm remains open to the present. The approach described above makes it possible to search for this optimal algorithm based on the methods of the optimal control theory.

The design trajectory behavior was analyzed for various initial values of state variables. The additional acceleration effect of the system design process was analysed by means of the variation of the initial value of the state variables and the special control functions. This effect exists owing to the very different behavior of the designing trajectories that have various control functions and different start points of the design space. It reduces the total computer time additionally and gives the perspective to accelerate more the system design process. The results obtained here serve as the first step for the optimal design algorithm characteristics determined, particularly for the initial point optimal selection and for the preliminary definition of the optimal trajectory and control functions structure.

The initial point selection permits obtain acceleration effect with a great probability. The trajectory analysis of various design strategies shows that the conception of the separatrix line or the separatrix hyper surface in general case is very helpful to understand and define the necessary and sufficient conditions for the design process acceleration effect existence. The separate hyper surface defines the start points and the trajectories that can produce the acceleration effect and can be used for the optimal designing trajectory construction. The selection of the initial points outside of the separatrix hyper surface is the necessary and sufficient conditions for the acceleration effect existence. The acceleration effect reduces the total computer time additionally and serves as the basis for the optimal or quasi-optimal algorithm construction.

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