ONLINE PARAMETRIC IDENTIFICATION OF MASS-SPRING-DAMPER MECHANICAL SYSTEMS USING ACCELERATION MEASUREMENTS

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Resumen

Implementación de esquemas de control activo de vibraciones, detección de fallas o tareas de monitoreo de la operación adecuada de estructuras mecánicas flexibles pueden requerir el uso de técnicas de identificación paramétrica ejecutadas en línea. Mediciones de señales de aceleración se usan en varias aplicaciones de identificación de parámetros en sistemas mecánicos vibratorios. En este artículo se propone un enfoque para estimación de parámetros en línea en el dominio del tiempo para sistemas mecánicos del tipo masa-resorte-amortiguador de *n* grados de libertad, usando únicamente mediciones de aceleración. Se usa integración por partes en la síntesis del método de identificación de parámetros propuesto. De esta manera, conocimiento previo de las condiciones iniciales del sistema son innecesarias. El método de estimación propuesto se puede extender para estimación paramétrica en tiempo real para sistemas mecánicos vibratorios no lineales, completamente actuados o sub-actuados. Se incluyen algunos resultados de simulación numérica para mostrar la efectividad del enfoque de estimación de parámetros de masa, rigidez y amortiguamiento, combinado con tareas de seguimiento de trayectorias de referencia en lazo-cerrado especificadas para el sistema mecánico vibratorio.

Palabras Claves: control activo de vibraciones, identificación de parámetros, sistemas mecánicos vibratorios, sistemas masa-resorte-amortiguador.

Abstract

Implementation of active vibration control schemes, failure detection and monitoring tasks of the suitable operation of flexible mechanical structures can require the use of on-line parametric identification techniques. Measurements of acceleration signals are preferred in several applications of parameter identification of vibrating mechanical systems. In this article, an on-line parameter estimation approach in time domain is proposed for linear mass-spring-damper mechanical systems of n degrees of freedom using acceleration measurements solely. Integration by parts is properly used in the synthesis of the proposed parameter identification method. In this fashion, a priori knowledge of the initial conditions of the system becomes unnecessary. The introduced identification method can be extended for real-time parametric estimation of nonlinear fully actuated or under-actuated nonlinear vibrating mechanical systems. Some numerical results are provided to show the effectiveness of the on-line estimation approach of the mass, stiffness and damping parameters combined with closed-loop reference trajectory tracking tasks specified for the vibrating mechanical system.

Keywords**:** *Active vibration control, mass-spring-damper systems, mechanical vibration systems, parameter identification.*

1. Introduction

Identification of vibration mechanical systems is an active research subject and its results admit several practical applications. Diverse methodologies have been mainly proposed for off-line estimation of parameters in the time domain or in the frequency domain [Soderstrom, 1989], [Isermann, 2011], [Ljung, 1987]. Some off-line estimation methods of modal parameters for mechanical systems are also described in [Heylen, 2003], [Le, 2013] and [Yang, 2013].

Recently, an on-line parametrical identification method for continuous-time constant linear systems has been proposed in [Fliess, 2003]. This algebraic approach is based on powerful mathematical tools of module theory, differential algebra and operational calculus. It is assumed that a mathematical model of the dynamic system is available for the synthesis of some parameter identifier/estimator. Thus, a suitable structure of a mathematical model describing the system dynamics is employed for the algebraic estimation of its parameters in a small time interval. Reasonably slow changes of some parameter values are also admitted during the system operation.

Algebraic identification has been successfully applied for on-line estimation of parameters and signals for vibrating mechanical systems using position measurements in [Beltran, 2015a], [Beltran, 2014], [Beltran, 2013] and [Beltran, 2013]. Harmonic forces can also be reconstructed on-line by employing algebraic system identification techniques [Beltran, 2015b]. Algebraic estimation of the frequency and amplitude of exogenous harmonic excitations in damped Duffing systems with an autoparametric pendulum vibration absorber has been introduced in [Silva, 2013].

This paper presents an on-line parameter estimation approach in continuous time domain for mass-spring-damper linear mechanical systems of n degrees of freedom using acceleration measurements solely. The presented results constitute a natural extension of previous works based on parameter identification using position measurements [Beltran, 2015a] and Mikusinski operational calculus [Mikusinski, 1983]. Integration by parts is properly used in the synthesis of the proposed parameter identification method. In this way, position and velocity measurements and priori knowledge of the initial conditions of the system are avoided. Thus, algebraic estimators for mass, stiffness and damping parameters can be reseated and updated continuously for operation scenarios where slow changes of the parameter values are expected. The introduced identification method can be extended for real-time parametric estimation of nonlinear fully actuated or under-actuated nonlinear vibrating mechanical systems. In fact, algebraic identification has been applied to sequentially estimate the parameters of nonlinear mass-spring-damper systems using position measurements [Beltran, 2014]. Some numerical simulation results are included to show the effectiveness of the on-line algebraic identification approach combined with adaptive-like reference trajectory tracking control for a Multiple-Input-Multiple-Output (MIMO) mechanical system of 3 degrees of freedom.

2. Methods

In the present section, the proposed algebraic parametric identification method for linear mass-spring-damper mechanical systems of *n* degrees of freedom using acceleration measurements is developed in detail.

Parametric Identification of a Mass-Spring-Damper System of *n* Degrees of Freedom

Firstly, consider the *n* DOF linear vibrating mechanical system schematically described in figure 1. Here, x_i, $i=1,2,\dots,n$, are the position coordinates, u_i the force control inputs, and m_i, k_i and c_i denote mass, stiffness and viscous damping parameters associated to the *i*-th DOF. The mathematical model of this MIMO flexible mechanical system is given by equation 1.

$$
\begin{aligned}
m_1\ddot{x}_1 + c_1\dot{x}_1 + k_1x_1 + k_2(x_1 - x_2) &= u_1 \\
m_2\ddot{x}_2 + c_2\dot{x}_2 + k_2(x_2 - x_1) + k_3(x_2 - x_3) &= u_2 \\
&\vdots \\
m_{n-1}\ddot{x}_{n-1} + c_{n-1}\dot{x}_{n-1} + k_{n-1}(x_{n-1} - x_{n-2}) + k_n(x_{n-1} - x_n) &= u_{n-1} \\
m_n\ddot{x}_n + c_n\dot{x}_n + k_n(x_n - x_{n-1}) + k_{n+1}x_n &= u_n
\end{aligned}
\qquad (1)
$$

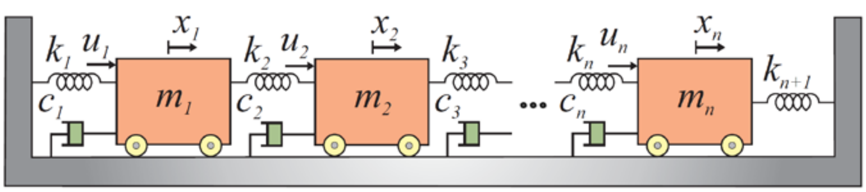

Figure 1 Schematic diagram of a *n* DOF mass-spring-damper system.

In the synthesis of the proposed parametric identification scheme, it is considered that only measurements of the acceleration output variables $y_i = \ddot{x}_i$ and the control inputs u_i are available. Therefore, Equation 1 are differentiated twice with respect to time and then multiplied by $\Delta^2 = (t - t_0)^2$ in order to avoid dependence on initial conditions of the system, where t_0 is the start time when the parameter identification process is performed, resulting equation 2.

$$\begin{aligned} m_1\Delta^2 x_1^{(4)} + c_1\Delta^2 x_1^{(3)} + k_1\Delta^2 \ddot{x}_1 + k_2\Delta^2\left(\ddot{x}_1 - \ddot{x}_2\right) &= \Delta^2\ddot{u}_1 \\ m_2\Delta^2 x_2^{(4)} + c_2\Delta^2 x_2^{(3)} + k_2\Delta^2\left(\ddot{x}_2 - \ddot{x}_1\right) + k_3\Delta^2\left(\ddot{x}_2 - \ddot{x}_3\right) &= \Delta^2\ddot{u}_2 \\ &\vdots \\ m_{n-1}\Delta^2 x_{n-1}^{(4)} + c_{n-1}\Delta^2 x_{n-1}^{(3)} + k_{n-1}\Delta^2\left(\ddot{x}_{n-1} - \ddot{x}_{n-2}\right) + k_n\Delta^2\left(\ddot{x}_{n-1} - \ddot{x}_n\right) &= \Delta^2\ddot{u}_{n-1} \\ m_n\Delta^2 x_n^{(4)} + c_n\Delta^2 x_n^{(3)} + k_n\Delta^2\left(\ddot{x}_n - \ddot{x}_{n-1}\right) + k_{n+1}\Delta^2\ddot{x}_n &= \Delta^2\ddot{u}_n \end{aligned} \tag{2}$$

Double integration by parts of equation 2 with respect to time yields to equation 3.

$$a_{11,i}(t)m_i + a_{12,i}(t)c_i + a_{13,i}(t)k_i + a_{14,i}(t)k_{i+1} = b_{1,i}(t),\ i = 1, 2, \ldots, n \tag{3}$$

With

$$\begin{aligned} a_{11,i} &= \Delta^2 y_i - 4\int_{t_0}^{t}\Delta y_1(\tau_1)\mathbf{d}\tau_1 + 2\int_{t_0}^{t}\int_{t_0}^{\tau_2} y_i(\tau_1)\mathbf{d}\tau_1\mathbf{d}\tau_2 \\ a_{12,i} &= \int_{t_0}^{t}\Delta^2 y_i\mathbf{d}\tau_1 - 2\int_{t_0}^{t}\int_{t_0}^{\tau_2}\Delta y_i(\tau_1)\mathbf{d}\tau_1\mathbf{d}\tau_2 \\ a_{13,i} &= \int_{t_0}^{t}\int_{t_0}^{\tau_2}\Delta^2\left[y_i(\tau_1) - y_{i-1}(\tau_1)\right]\mathbf{d}\tau_1\mathbf{d}\tau_2 \\ a_{14,i} &= \int_{t_0}^{t}\int_{t_0}^{\tau_2}\Delta^2\left[y_i(\tau_1) - y_{i+1}(\tau_1)\right]\mathbf{d}\tau_1\mathbf{d}\tau_2 \\ b_{1,i} &= \Delta^2 u_i - 4\int_{t_0}^{t}\Delta u_i(\tau_1)\mathbf{d}\tau_1 + 2\int_{t_0}^{t}\int_{t_0}^{\tau_2} u_i(\tau_1)\mathbf{d}\tau_1\mathbf{d}\tau_2 \end{aligned} \tag{4}$$

Where $y_i = \ddot{x}_i$, and $y_0 = y_{n+1} \equiv 0$.

Equation 3, after three more integrations, leads to the linear system of equations 5.

$$\theta_i = A_i^{-1}B_i = \frac{1}{\Delta_i}\left[\Delta_{1,i} \quad \Delta_{2,i} \quad \Delta_{3,i} \quad \Delta_{4,i}\right]^T,\ i = 1, 2, \ldots, n \tag{5}$$

Where $\theta_i = \left[m_i, c_i, k_i, k_{i+1}\right]^T$ is the vector of positive constant parameters to be identified, A_i and B_i are 4×4 and 4×1 matrices, respectively, described by

$$A_i = \begin{bmatrix} a_{11,i} & a_{12,i} & a_{13,i} & a_{14,i} \\ a_{21,i} & a_{22,i} & a_{23,i} & a_{24,i} \\ a_{31,i} & a_{32,i} & a_{33,i} & a_{34,i} \\ a_{41,i} & a_{42,i} & a_{43,i} & a_{44,i} \end{bmatrix}, \; B_i = \begin{bmatrix} b_{1,i} \\ b_{2,i} \\ b_{3,i} \\ b_{4,i} \end{bmatrix}$$

Whose components are considered as output signals of the dynamic system, equation 6.

$$\begin{aligned} \dot{a}_{kh,i} &= a_{k-1h,i} \\ \dot{b}_{k,i} &= b_{k-1,i} \end{aligned} \tag{6}$$

With $i = 1, 2, \ldots, n$, $k = 2, 3, 4$, $h = 1, 2, 3, 4$, and zero initial conditions at $t = t_0$.

Hence, the estimators (equation 7) are proposed for the algebraic estimation of the mass, damping and stiffness parameters using measurements of acceleration signals y_i:

$$\begin{aligned} \widehat{m}_i &= \frac{\int_{t_0}^{(2)} |\Delta_{1,i}|}{\int_{t_0}^{(2)} |\Delta_i|}, \quad \widehat{c}_i = \frac{\int_{t_0}^{(2)} |\Delta_{2,i}|}{\int_{t_0}^{(2)} |\Delta_i|} \\ \widehat{k}_i &= \frac{\int_{t_0}^{(2)} |\Delta_{3,i}|}{\int_{t_0}^{(2)} |\Delta_i|}, \quad \widehat{k}_{i+1} = \frac{\int_{t_0}^{(2)} |\Delta_{4,i}|}{\int_{t_0}^{(2)} |\Delta_i|}, \; \forall t > t_0 > 0 \end{aligned} \tag{7}$$

Where $\widehat{(\cdot)}$ denotes estimated parameter and $\int_{t_0}^{(2)} \phi(t)$ the iterated integral of the form $\int_{t_0}^{t}\int_{t_0}^{\tau_2} \phi(\tau_1) \mathbf{d}\tau_1 \mathbf{d}\tau_2$.

3. Results

Main results of the present contribution are constituted by the algebraic formulas or estimators (7) to compute mass, stiffness and damping parameters in flexible mechanical systems using measurements of acceleration signals.

In this section, some numerical simulation results are included to depict the effectiveness of the parameter identification approach on a 3 DOF MIMO mechanical system characterized by the set of parameters described in table 1.

Table 1 Parameters of the vibrating mechanical system.

Mass (kg)	Damping (N.s/m)	Stiffness (N/m)
m_1 = 2.0	c_1 = 5.5	k_1 = 1000
m_2 = 2.5	c_2 = 5.0	k_2 = 900
m_3 = 3.0	c_3 = 4.5	k_3 = 900
		k_4 = 700

The main interest of the present work resides on the fast identification of the system parameters. Thus, constant or variable forces u_i can be applied to the mechanical system to get estimates of the mass, damping and stiffness parameters in a small period of time. Nevertheless, the output feedback control scheme proposed in [Beltran, 2015a] was used to assess the dynamic performance of the estimation approach for reference trajectory tracking tasks, equations 8 a la 10.

$$u_i = \widehat{m}_i v_i + \widehat{c}_i \widehat{\dot{x}}_i + \widehat{k}_i (x_i - x_{i-1}) + \widehat{k}_{i+1}(x_i - x_{i+1}) \tag{8}$$

With

$$v_i = \ddot{y}_i^* - \beta_{2,i}(\widehat{\dot{x}}_i - \dot{x}_i^*) - \beta_{1,i}(x_i - x_i^*) - \beta_{0,i}\int_0^t (x_i - x_i^*)\mathbf{d}t \tag{9}$$

$$\begin{aligned} \widehat{\dot{x}}_i = & -\frac{\widehat{c}_i}{\widehat{m}_i}x_i - \frac{\widehat{k}_i}{\widehat{m}_i}\int_0^t (x_i - x_{i-1})\mathbf{d}t - \frac{\widehat{k}_{i+1}}{\widehat{m}_i}\int_0^t (x_i - x_{i+1})\mathbf{d}t \\ & + \frac{1}{\widehat{m}_i}\int_0^t u_i \mathbf{d}t \end{aligned} \tag{10}$$

Where i =1,2,3.

Figures 2 and 3 display the accurate and fast estimation of the mass, stiffness and damping parameters using the proposed algebraic formulas 7. The satisfactory tracking of position reference trajectories planned for the vibration mechanical system is clearly manifested as well. In the next section some important discussions about the analytical and numerical results are described.

4. Discussion

A very good closed-loop estimation of the system parameters using the algebraic estimators 7 and control forces 8 was verified in figure 2. At the beginning, in the control implementation the mass values were fixed at 1 kg and the other parameter values at 0. Next, at $t > 0.1$ the estimates were replaced in the tracking controllers. The Runge-Kutta-Fehlberg 4/5 method with step time of 1 x 10^{-3} s was used in the numerical tests. Hence, accurate and fast estimates of the mass, damping and stiffness parameters were computed before 0.1 s. Thus, parameter estimators can be reseated and updated continuously for operation scenarios where slow changes of the parameter values are expected.

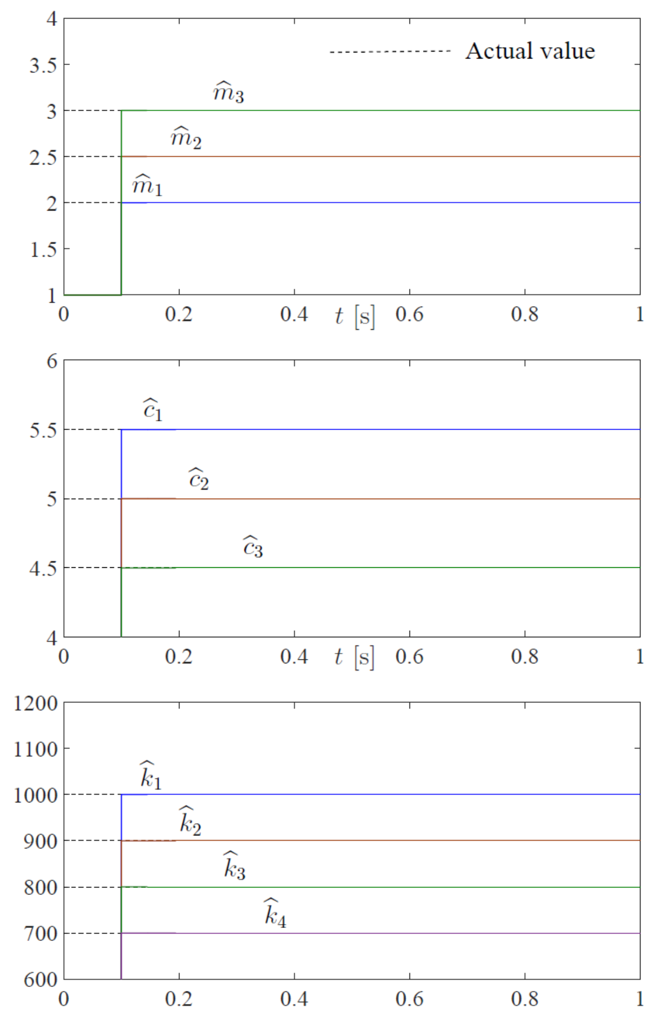

Figure 2 Closed-loop algebraic estimation of mass, damping and stiffness parameters.

A satisfactory closed-loop tracking of the reference trajectories using the estimates of the mechanical system parameters is displayed in figure 3. The desired motion profiles are described by Bézier interpolation polynomials which were defined to firstly transfer the mechanical system from a rest equilibrium state to another for

$\bar{x}_1 = 0.005$ m, $\bar{x}_2 = 0.005$ m and $\bar{x}_3 = -0.005$, and next to the rest equilibrium state again in 5 s. The controlled forces applied to the vibration system are shown in figure 4. A reasonable momentary overshoot in the force responses is presented when the estimated parameters are substituted in the control algorithms (8) at $t > 0.1$ s.

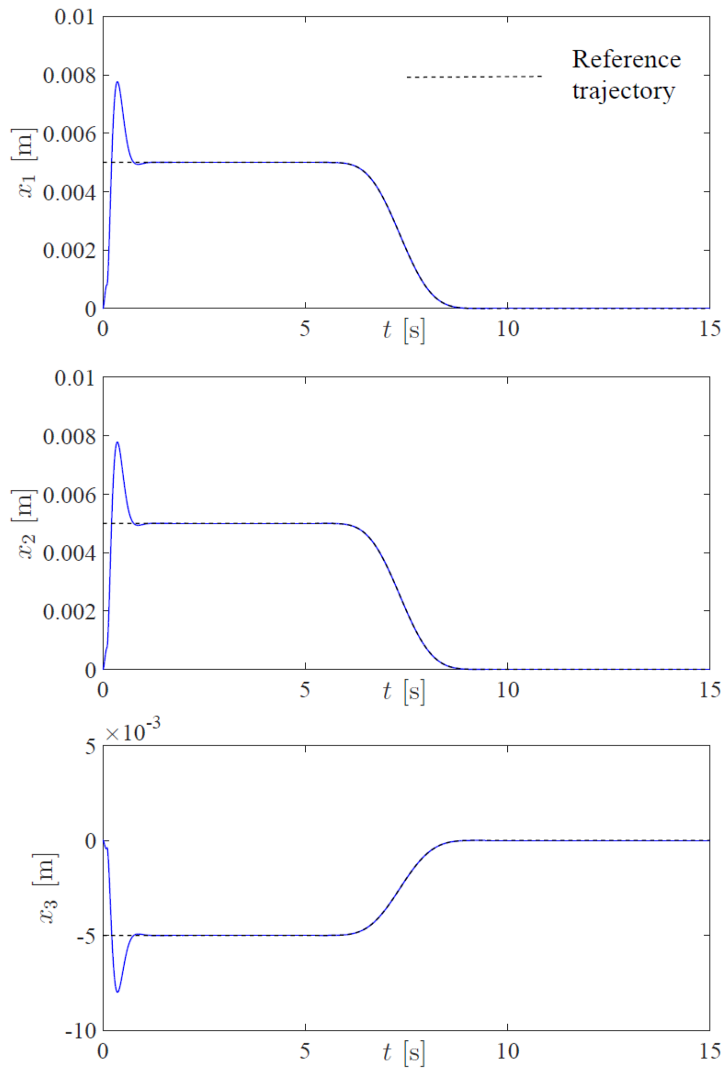

Figure 3 Control forces applied to the mechanical system for trajectory tracking tasks.

Therefore, the analytical and numerical results have confirmed the effectiveness of the on-line estimation approach of mass, stiffness and damping parameters combined with closed-loop reference trajectory tracking tasks on multiple-input multiple-output vibrating mechanical system of *n* degrees of freedom.

5. Conclusions

An algebraic identification method for mass, damping and stiffness parameters for linear MIMO mechanical systems of *n* degrees of freedom using acceleration measurements has been proposed. The presented identification approach constitutes an extension of the parametric identification method for linear mass-spring-damper mechanical system using position measurements introduced in [Beltran, 2015a]. The parameter values are estimated accurately and algebraically into a small windows of time depending on the numerical integration method and the velocity and precision of the computer processor employed for the implementation of the algebraic estimators. The dynamic performance of the proposed parameter estimators was numerically evaluated for closed-loop tracking tasks of reference trajectories on a 3 DOF MIMO mechanical system. Preliminary computer simulation results show a good estimation of the unknown parameters of the linear vibration mechanical system. Future studies will include the application of the algebraic identification approach to the on-line parametric estimation problem of nonlinear vibrating mechanical systems.

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