MAXIMUM PRINCIPLE FOR TIME MINIMIZATION OF CIRCUIT DESIGN PROCESS

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Resumen

Se analiza la posibilidad de aplicar el principio máximo de Pontryagin al problema de la optimización de circuitos electrónicos. Se demuestra que a pesar de que el problema de la optimización es formulado como una tarea no lineal, y el principio máximo en este caso no es una condición suficiente para obtener mínimo del funcional, es posible obtener la decisión en la forma de mínimos locales. La aceleración relativa del tiempo de cómputo para la mejor estrategia encontrada por medio del principio máximo comparado con el enfoque tradicional es igual de dos a tres órdenes de magnitud.

Palabras Claves: Estrategias de optimización, optimización del circuito, principio máximo de Pontryagin, sistema dinámico controlable.

Abstract

The possibility of applying the maximum principle of Pontryagin to the problem of optimization of electronic circuits is analysed. The presented theoretical approach is directed to a possibility of designing of any analog circuits. It is shown that in spite of the fact that the problem of optimization is formulated as a nonlinear task, and the maximum principle in this case isn't a sufficient condition for obtaining a maximum of the functional, it is possible to obtain the decision in the form of local minima. The relative acceleration of the CPU time for the best strategy found by means of maximum principle compared with the traditional approach is equal two to three orders of magnitude.

Keywords**:** *Circuit optimization; controllable dynamic system; optimization strategies; maximum principle of Pontryagin.*

1. Introduction

To improve the overall quality of electronic circuit designs, it is very important to reduce their design time. Many works devoted to this problem focus on how to reduce the number of operations when solving two main problems: circuit analysis and numerical optimization. By solving these problems successfully, one can reduce the total time required for analog circuit optimization and this fact serves as a basis for improving design quality. The methods used to analyse complex systems are being improved continuously. Some well-known ideas related to the use of a method of sparse matrixes [Osterby, 1983] and decomposition methods [Rabat et al., 1985] are used for the reduction of time for the analysis of circuits. Some alternative methods such as homotopy methods [Tadeusiewicz, 2013] were successfully applied to circuit analysis.

Different techniques for analog circuit optimization can be classified in two main groups: deterministic optimization algorithms and stochastic search algorithms.

Practical methods of optimization were developed for circuit designing, timing, and area optimization [Brayton et al., 1987]. However, classical deterministic optimization algorithms may have a number of drawbacks: they may require that a good initial point be selected in the parameter space, they may reach an unsatisfactory local minimum, and they require that the cost function be continuous and differentiable. To overcome these issues, special methods were applied to determine the initial point of the process by centering [Stehr et al., 2003] or applying geometric programming methods [Hershenson et al., 2001].

Stochastic search algorithms, especially evolutionary computation algorithms like genetic algorithms, differential evaluation, genetic programming, particle swarm optimization, etc. have been developed in recent years [Alpaydin et al., 2003], [Srivastava et al., 2007], [Liu et al., 2009], [Yengui et al., 2012]. Genetic algorithms have been employed as optimization routines for analog circuits due to the ability to find a satisfactory solution. A special algorithm defined as a particle swarm optimization technique is one of the evolutionary algorithms and competes with genetic algorithms. This method is successfully used for electromagnetic problems and for optimization of microwave systems [Robinso, 2004], [Ridzuan et al., 2016].

A more general formulation of the circuit optimization problem for deterministic approach was developed on a heuristic level some decades ago [Kashirskiy, 1979]. This approach ignored Kirchhoff's laws for all or part of a circuit during the optimization process. The practical aspects of this idea were developed for the optimization of microwave circuits [Rizzoli et al., 1990] and for the synthesis of high-performance analog circuits [Ochotta et al., 1996] in an extreme case where all the equations of the circuit were not solved during the optimization process.

In work [Zemliak, 2001] the problem of circuit optimization is formulated in terms of the theory of optimal control. Thus, the process of circuit optimization was generalized and defined as the dynamic controllable system. In this case, the basic element is the control vector that changes the structure of the equations of the system of optimization process. Thus, there is a set of strategies of optimization that have different number of operations and different computing times. The introduction and analysis of the function of Lyapunov of the optimization process [Zemliak, 2008], [Zemliak, 2015] allows comparison of various strategies of optimization and choosing the best of them having minimum processor time. At the same time, the problem of searching for the optimal strategy and the corresponding optimal trajectory can be solved most appropriately within the maximum principle of Pontryagin [Pontryagin et al., 1962].

The main complexity of application of the maximum principle consists of the search of initial values for auxiliary variables at the solution of the conjugate system of equations. Application of the maximum principle in case of linear dynamic systems is based on the creation of an iterative process [Neustadt, 1960], [Rosen, 1966].

In case of nonlinear systems, the convergence of this process is not guaranteed. However, application of the additional approximating [Bourdin, 2013] allows constructing sequence of the solutions converging to a limit under certain conditions.

The first step in the problem of possibility of application of maximum principle for circuit optimization was presented briefly in [Zemliak, 2016]. In the present work, the analysis of the problem is presented in detail in section 2 for two-dimensional case and the numerical results are presented in section 3.

2. Methods

Let's analyse an example of the optimization of the elementary nonlinear circuit for which the solution was obtained on the basis of the maximum principle. We will consider the simplest nonlinear circuit of a voltage divider, figure 1.

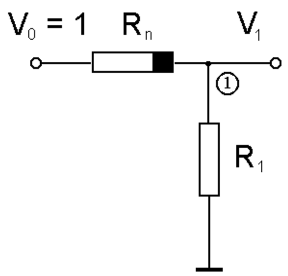

Figure 1 Simplest nonlinear voltage divider.

Let us consider that the nonlinear element has the following equation 1.

$$R_n = a + b(V_1 - V_0) \quad (1)$$

Where *a*>0, *b*>0, *a*>*b*, V_o and V_1 the voltages on an input and an output of circuit. We will consider that V_o is equal 1. We will define the variables x_1, x_2. $x_1 = R$, $x_2 = V_1$. Thus the vector of phase variables $X \in R^2$. In this case the formula 1 can be replaced with the following equation 2.

$$R_n = a + b(x_2 - 1) \quad (2)$$

We can present the equation 3 of a circuit in the form:

$$g_1(x_1, x_2) \equiv x_2[x_1 + a + b(x_2 - 1)] - x_1 = 0 \quad (3)$$

The circuit optimization is formulated as a problem of obtaining at the exit of a circuit of the defined voltage *w*. We will determine the cost function of the optimization process by the equation 4.

$$C(X) = (x_2 - w)^2 \quad (4)$$

In this case, the problem of circuit optimization is converted to minimization of the cost function $C(X)$. Following theoretical bases that were developed in [Zemliak, 2007] we formulate the problem for circuit optimization as a task of search of the optimization strategy with a minimum possible CPU time. For this purpose, we define the functional, which is subject to minimization, by the equation 5.

$$J = \int_0^T f_0(X)\, dt \tag{5}$$

Where $f_0(X)$ is the function that is conditionally determining the density of a number of arithmetic operations in a unit of time *t*. In that case, integral (equation 5) defines total number of operations necessary for circuit optimization and is proportional to the total CPU time.

The structure of function $f_0(X)$ cannot be defined. However, we can compute CPU time using the possibilities of the compiler. We will further identify the integral with CPU time, and therefore, the problem of minimization of CPU time corresponds to a problem of minimization of the integral.

According to [Zemliak, 2001], we introduce the control vector *U* that consists of only one component *u(t)* for the reviewed example. This component has one of two possible values: 0 or 1. The control vector allows to generalise circuit optimization process and to define a set of the optimization strategies differing in operations number and CPU time. The generalized cost function is defined in this case by the equation 6.

$$F(X) = C(X) + \varphi(X) \tag{6}$$

Where $\varphi(X)$ is an additional penalty function, which can be determined, for example, by the equation 7.

$$\varphi(X) = \sum_{j=1}^{M} u_j \cdot g_j^2(X) \tag{7}$$

Where *M* is the number of nodes of the circuit. In our case *M*=1.

Process of circuit optimization thus can be described by the system (equation 8) with restrictions (equation 9).

$$\frac{dx_i}{dt} = f_i(x_1, x_2, u), \qquad i=1,\ 2 \tag{8}$$

$$(1-u)\cdot g_1(x_1,x_2)=0 \tag{9}$$

Where functions $f_i(x_1,x_2,u)$ are defined by a concrete numerical method of optimization. When using a gradient method, these functions are defined by the equation 10.

$$f_i(x_1,x_2,u)=-\frac{\delta}{\delta x_i}F(X),\ i=1,2 \tag{10}$$

Where the operator $\delta/\delta x_i$ is defined by the expression:

$$\frac{\delta}{\delta x_i}\sigma(X)=\frac{\partial\sigma(X)}{\partial x_i}+\sum_{p=K+1}^{K+M}\frac{\partial\sigma(X)}{\partial x_p}\frac{\partial x_p}{\partial x_i}$$

The value *u(t)*=0 corresponds to the traditional strategy of optimization (TSO). In this case in the system (8), there is only one equation for the independent x_1 variable, whereas the variable x_2 is defined from the equation 9. The value *u(t)*=1 corresponds to the modified traditional strategy of optimization (MTSO) when both x_1 and x_2 variables are independent. In this case, the system (8) includes two equations for the independent variables x_1 и x_2, and the equation 9 disappears. A change in the value of function *u(t)* with 0 on 1 and back can be made at any moment and generates a set of various strategies of optimization. Two main strategies are defined as follows:

- TSO, *u*=0. The equations 8 to 10 are replaced with the following equations 11 y 12.

$$\frac{dx_1}{dt}=-\frac{\partial C}{dx_2}\frac{dx_2}{dx_1} \tag{11}$$

$$\frac{dx_2(x_1,t)}{dt}=\frac{\partial x_2}{\partial x_1}\frac{dx_1}{dt} \tag{12}$$

Where the derivative dx_2/dx_1 is defined from the equation 9 and can be calculated by the formula:

$$\frac{dx_2}{dx_1}=\frac{1}{2b}\left[-1+\frac{x_1+c+2b}{\sqrt{(x_1+c)^2+4bx_1}}\right],\ c=a-b$$

- MTSO, *u*=1. The equations 8 are transformed to the next one:

$$\frac{dx_i}{dt} = -\frac{\delta}{\delta x_i}\left[C(X) + g_1^2(X)\right] \qquad i=1,\,2 \tag{13}$$

In a general case, the right-hand parts of the equations 8 can be presented in equation 14.

$$\begin{aligned} f_1(x_1,x_2,u) &= (1-u)\cdot f_{11}(x_1,x_2) + u\cdot f_{12}(x_1,x_2) \\ f_2(x_1,x_2,u) &= (1-u)\cdot f_{21}(x_1,x_2) + u\cdot f_{22}(x_1,x_2) \end{aligned} \tag{14}$$

Where the functions $f_{ij}(x_1,x_2)$ are determined by the following equations 15.

$$\begin{aligned} f_{11}(x_1,x_2) &= \frac{(w-x_2)}{b}\left[-1 + \frac{x_1+c+2b}{\sqrt{(x_1+c)^2+4bx_1}}\right] \\ f_{12}(x_1,x_2) &= -2(x_2-1)\{(x_2-1)x_1 + [a+b(x_2-1)]x_2\} \\ f_{21}(x_1,x_2) &= \frac{(w-x_2)}{2b^2}\left[-1 + \frac{x_1+a+b}{\sqrt{(x_1+c)^2+4bx_1}}\right]^2 \\ f_{22}(x_1,x_2) &= -2(x_2-w) - 2(c+x_1+2bx_2)\cdot[(x_2-1)x_1 + ax_2 + b(x_2-1)x_2] \end{aligned} \tag{15}$$

According to methodology of the maximum principle, the system of the conjugate equations for additional variables ψ_1, ψ_2 has the next equations 16.

$$\begin{aligned} \frac{d\psi_1}{dt} &= -\frac{\partial f_1(x_1,x_2,u)}{\partial x_1}\cdot\psi_1 - \frac{\partial f_2(x_1,x_2,u)}{\partial x_1}\cdot\psi_2 \\ \frac{d\psi_2}{dt} &= -\frac{\partial f_1(x_1,x_2,u)}{\partial x_2}\cdot\psi_1 - \frac{\partial f_2(x_1,x_2,u)}{\partial x_2}\cdot\psi_2 \end{aligned} \tag{16}$$

Where partial derivatives of the functions $f_i(x_1,x_2,u)$, *i*=1, 2 are calculated by the next equations 17.

$$\frac{\partial f_{11}(x_1,x_2)}{\partial x_1} = \frac{(x_2-w)4a}{\left[(x_1+c)^2+4bx_1\right]^{3/2}}$$

$$\frac{\partial f_{12}(x_1,x_2)}{\partial x_1}=-2(x_2-1)^2$$

$$\frac{\partial f_{21}(x_1,x_2)}{\partial x_1}=-\frac{(w-x_2)}{b}\left[-1+\frac{x_1+a+b}{\sqrt{(x_1+c)^2+4bx_1}}\right]\frac{4a}{\left[(x_1+c)^2+4bx_1\right]^{3/2}}$$

$$\frac{\partial f_{22}(x_1,x_2)}{\partial x_1}=-2(x_2-1)x_1-2\left[a+b(x_2-1)\right]x_2-2(c+x_1+2bx_2)(x_2-1)$$

(17)

$$\frac{\partial f_{11}(x_1,x_2)}{\partial x_2}=-\frac{1}{b}\left[-1+\frac{x_1+a+b}{\sqrt{(x_1+c)^2+4bx_1}}\right]$$

$$\frac{\partial f_{12}(x_1,x_2)}{\partial x_2}=-4(x_2-1)x_1-2\left[a+b(x_2-1)\right]x_2-2(x_2-1)\left[bx_2+a+b(x_2-1)\right]$$

$$\frac{\partial f_{21}(x_1,x_2)}{\partial x_2}=-\frac{1}{2b^2}\left[-1+\frac{x_1+a+b}{\sqrt{(x_1+c)^2+4bx_1}}\right]^2$$

$$\frac{\partial f_{22}(x_1,x_2)}{\partial x_2}=-2-4b\left[(x_2-1)x_1+bx_2^2+cx_2\right]-2(c+x_1+2bx_2)^2$$

The Hamiltonian is expressed by the following equation 18.

$$H=\psi_1\cdot f_1(x_1,x_2,u)+\psi_2\cdot f_2(x_1,x_2,u) \tag{18}$$

Substituting equation 14 in 18 and doing identical transformations, we obtain the following expression for the Hamiltonian, equation 19.

$$H=\psi_1\cdot f_{11}(x_1,x_2)+\psi_2\cdot f_{21}(x_1,x_2)+u\cdot\Phi(x_1,x_2,\psi_1,\psi_2) \tag{19}$$

Where

$$\Phi(x_1,x_2,\psi_1,\psi_2)=\psi_1\cdot\left[f_{12}(x_1,x_2)-f_{11}(x_1,x_2)\right]+\psi_2\cdot\left[f_{22}(x_1,x_2)-f_{21}(x_1,x_2)\right]$$

According to the maximum principle, we obtain the next main condition for the control function *u*:

$$u=\begin{cases}0, & \Phi<0\\ 1, & \Phi>0\end{cases} \tag{20}$$

The behaviour of the control function *u(t)* that corresponds to the maximum principle is also defined by the behaviour of functions $\psi_1(t)$ and $\psi_2(t)$, which are computed from the equations 16.

3. Results

Numerical results were obtained on computer Sony VAIO PCG-V505MFP, Windows XP, processor Pentium 4-M, 2.2 GHz with compiler C++. The solution of the equations 16 depends on the initial values ψ_{10} and ψ_{20}, which are defined within the precision of the common multiplier. One of these constants can be taken arbitrarily. Let us define the constant $\psi_{10} = -1$. The value of the constant ψ_{20}, which corresponds to the correct solution of a task in the conditions of the maximum principle ψ_{20c} , can be obtained by iterative procedure. We use the iterative procedure on the basis of the gradient method, which minimise the functional (5). The minimum value of this functional can be provided by the correct value of parameter ψ_{20c}.

The analysis of the process of optimization for a similar example, which is carried out in work [Zemliak, 2002], showed that the TSO (*u*=0) is the optimal one when both initial values of variables x_1 and x_2, (x_{10}, x_{20}) are positive. In this case the number of iterations is equal to 3898, and CPU time is equal to 42.88 msec for the initial point x_{10} =1, x_{20} =2. At the same time, the negative initial values of the variable x_2 significantly lead to other results. In the case of negative initial values of the variable x_2, emergence of effect of acceleration of the process of circuit optimization is possible [Zemliak, 2002]. This effect accelerates the optimization process in many times. It is interesting to check if this result corresponds to the maximum principle, figure 2 shows the trajectories of the process of circuit optimization with the negative initial value of coordinate x_{20}, (x_{10} =1, x_{20} = –2). The structure of function *u(t)* that was obtained automatically and corresponds to a condition of the maximum principle (20) has one or two points of a rupture that corresponds to switching from the trajectory corresponding to MTSO (*u*=1, a dotted curve) on trajectory corresponding to TSO (*u*=0, a continuous curve). Coordinates of a switching point of t_{sw} depend on the value of Ψ_{20}. The data corresponding to the different points of switching from 1 to 11 in figure 2 are presented in table 1.

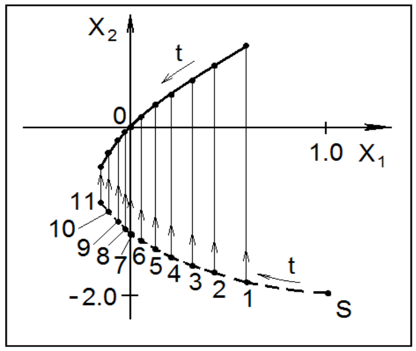

Figure 2 Trajectories of optimisation process with initial point (X_{10} =1, X_{20} = –2) and different values of Ψ_{20}.

Table 1 Data of some strategies with different initial values of variable $\psi_2(t)$.

N	ψ_{20}	Control function structure	Swiching points	Total iterations number	CPU time (msec)
1	7.27	1; 0; 1	198; 199	2606	14.34
2	7.265	1; 0; 1	200; 201	2464	13.56
3	7.26	1; 0; 1	202; 203	2274	12.52
4	7.255	1; 0; 1	203; 204	2148	11.82
5	7.25	1; 0; 1	205; 206	1759	9.68
6	7.245	1; 0	206	207	1.14
7	7.24	1; 0	209	620	5.67
8	7.235	1; 0	211	711	6.66
9	7.23	1; 0	214	785	7.46
10	7.225	1; 0	216	818	7.81
11	7.22	1; 0	219	855	8.21

A change in the value of Ψ_{20} from 7.27 to 7.245 leads to reduction of iterations number and CPU time from 14.34 to 1.14 ms, but the CPU time is increasing later on. That is visible also in figure 3, where the dependence of CPU time of the solution of a task from initial value Ψ_{20} is shown.

The value ψ_{20opt} = 7.245 corresponds to the minimum CPU time T_{min} and in this case the integral *J* and the initial value of variable $\psi_2(t)$ provides the maximum value of a Hamiltonian according to the maximum principle. The gain in time computed as time relation for TSO by the minimum time of T_{min} thus equal to 37.6 times.

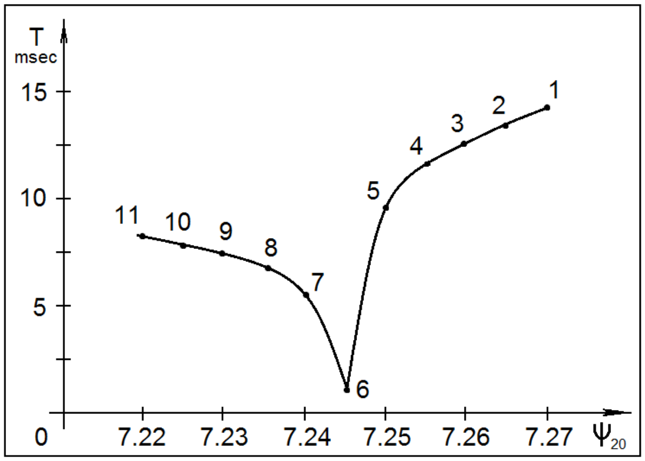

Figure 3 CPU time for different initial values of variable $\psi_2(t)$.

Let us define the partial Hamiltonians $H_{(0)}$, $H_{(1)}$ by the equations 21 y 22.

$$H_{(0)} = \psi_1 \cdot f_1(x_1, x_2, 0) + \psi_2 \cdot f_2(x_1, x_2, 0) \tag{21}$$

$$H_{(1)} = \psi_1 \cdot f_1(x_1, x_2, 1) + \psi_2 \cdot f_2(x_1, x_2, 1) \tag{22}$$

Dependencies of the functions $H_{(0)}(t)$, $H_{(1)}(t)$ and $\Phi(t)$ for various values of parameter Ψ_{20} are presented in figures 4, 5 y 6. Optimum value of a constant Ψ_{20} is equal to 7.245 and corresponds to the results presented in figure 4.

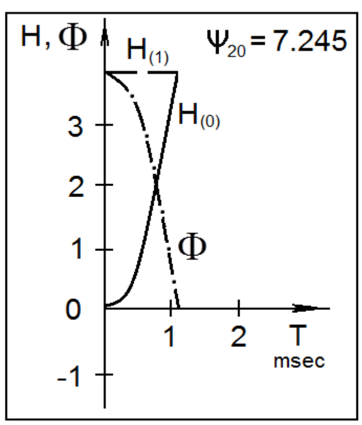

Figure 4 Time dependency of functions $H_{(0)}(t)$, $H_{(1)}(t)$ and $\Phi(t)$ for optimal parameter Ψ_{20}.

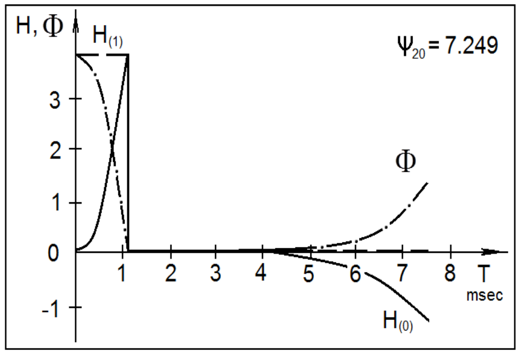

Figure 5 Time dependency of functions $H_{(0)}(t)$, $H_{(1)}(t)$ and $\Phi(t)$ for non-optimal value of parameter Ψ_{20} , $\Psi_{20} > \Psi_{20opt}$.

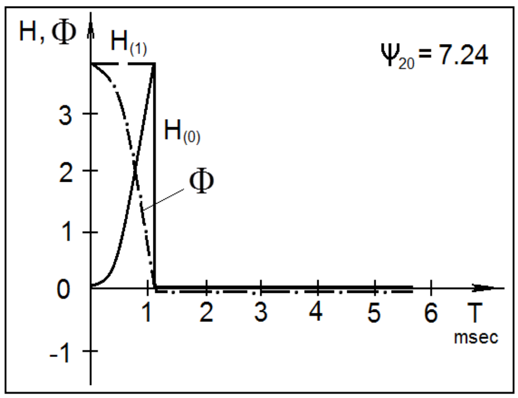

Figure 6 Time dependency of functions $H_{(0)}(t)$, $H_{(1)}(t)$ and $\Phi(t)$ for non-optimal value of parameter Ψ_{20} ,$\Psi_{20} < \Psi_{20opt}$.

In this case the function $H_{(1)}(t)$ passes above the function $H_{(0)}(t)$ from the beginning of the process until the point T_{sw}. At this point both functions become equal, function $\Phi(t)$ changes a sign, and according to condition (20), value of the control function *u* is changing to 1 on 0. Then, the iterative process comes to the end because the criterion for the end of the optimization process is satisfied.

It is interesting to analyse the behaviour of the functions $H_{(0)}(t)$, $H_{(1)}(t)$ and $\Phi(t)$ with non-optimal initial value Ψ_{20} . In this case, the point of switching the control function *u* from 1 on 0 is not the optimal one. The behaviour of functions $H_{(0)}(t)$, $H_{(1)}(t)$ and $\Phi(t)$ is shown in figure 5 for other initial value of Ψ_{20} =7.249 that is greater than the optimal one.

In this case the control function switching happens before an optimum point and the time of computing grows till 7.55 msec.

The behaviour of these functions is given in figure 6 for the initial value of ψ_{20} =7.24 that is lesser than the optimal one. In this case the control function switching happens after an optimum point and the time of computing grows again to 5.67 msec.

We can see that in this case the optimization process is longer than for the optimal value of the parameter ψ_{20} .

4. Discussion

The application of the maximum principle gives us the possibility to find the optimal structure of the control vector U. It means that the main goal of the problem of optimal control theory is achieved on the basis of maximum principle.

It is clear that when the point of switching of the control vector differs from the optimal one, the value of the Hamiltonian is changing over time. On the other hand the Hamiltonian has a permanent when the optimal position of switch point is applied. It was obtained two principal results. First, the theoretical justification is given for the earlier discovered effect of acceleration of the process of circuit optimization in the conditions of a new methodology of design. This justification is based on the maximum principle. Second, the analysis of the optimization process for the analysed circuit has shown that application of the maximum principle really allows for the finding of the optimum structure of the control vector U(t) by means of the iterative procedure. We found this structure automatically on the basis of the main principle (20). Besides, the considerable reduction of the processor time in comparison with the traditional approach is observed when using the maximum principle.

5. Conclusions

The analysis of optimization process of the presented circuit showed that application of the maximum principle really allows finding the optimum structure of the control vector U(t) by means of iterative procedure. Thus, considerable reduction of CPU time in comparison with traditional approach is observed.

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